Cosmological Data Analysis of the Power-Law $f(Q)$ Gravity Model Using Recent Datasets

Praveen Kumar Dhankar[1], Pawan Kumar Verma[2], Mayur Mune[3], Safiqul Islam[4]

[1,2]Lincoln University College, Petaling Jaya, Malaysia

[1,2,3]Symbiosis Institute of Technology, Nagpur Campus, Symbiosis International (Deemed University), Pune 440008, Maharashtra, India

[4]Department of Mathematics and Statistics, College of Science, King Faisal University, P.O. Box 400, Al Ahsa 31982, Saudi Arabia.

Email ID: pdf.praveendhankar@lincoln.edu.my[1], pkumar6743@gmail.com[1], abes.pawan@gmail.com[2], mayurmune1999@gmail.com[3] sislam@kfu.edu.sa[4]

Abstract: The accelerated expansion of the Universe has motivated extensive studies of modified theories of gravity as alternatives to dark energy. In this work, we investigate the power-law $f(Q)$ gravity model in the framework of symmetric teleparallel gravity using recent cosmological observations. The model parameters are constrained through a Bayesian Markov Chain Monte Carlo analysis employing two combinations of datasets: Pantheon+SHOES + Hubble measurements and Pantheon+SHOES + Hubble measurements + DESI baryon acoustic oscillation data. The modified Friedmann equation corresponding to the power-law $f(Q)$ model is solved numerically to reconstruct the cosmic expansion history. The resulting constraints are compared with those obtained for the standard ΛCDM cosmology. Model performance is assessed using the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC). This analysis provides updated observational constraints on power-law $f(Q)$ gravity and examines its consistency with current cosmological observations.

Keywords: $f(Q)$ gravity, Numerical Methods, MCMC.

Introduction

Observations of Type Ia supernovae have provided compelling evidence that the Universe is currently undergoing an accelerated phase of expansion [1,2]. The standard ΛCDM model successfully describes a broad range of cosmological observations and remains the most widely accepted cosmological framework. Nevertheless, the physical nature of the cosmological constant remains uncertain, motivating the search for alternative explanations of the late-time acceleration of the Universe.

Among the various approaches proposed in the literature, modified theories of gravity offer a geometric explanation of cosmic acceleration without introducing an additional dark energy component. Within the framework of Symmetric Teleparallel Gravity, gravitation is described by the non-metricity scalar Q, rather than curvature or torsion. This formulation leads naturally to $f(Q)$ gravity, where the gravitational action is generalized from the non-metricity scalar to an arbitrary function of Q [3,4]. Owing to its rich cosmological phenomenology and mathematical simplicity, $f(Q)$ gravity has attracted considerable attention in recent years.

Several forms of $f(Q)$ gravity have been investigated using cosmological observations. In particular, the power-law model has emerged as one of the simplest extensions of the symmetric teleparallel equivalent of General Relativity and has been shown to provide viable cosmological solutions [5,6].

In this work, we revisit the power-law $f(Q)$ gravity model using Pantheon+ SHOES, Hubble and DESI BAO data. The model parameters are constrained through a Bayesian Markov Chain Monte Carlo analysis and compared with the standard ΛCDM cosmology using Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC) statistics.

Related work and Key Contribution

Lazkoz et al. [5] investigated observational constraints on $f(Q)$ gravity using multiple cosmological probes and showed that non-metricity-based gravity can successfully describe the late-time expansion history of the Universe. More recently, Mhamdi et al. [6] constrained the power-law and exponential $f(Q)$ models using Pantheon+, $H(z)$, and RSD data and found that the exponential model is statistically preferred. Motivated by these studies, we revisit the power-law $f(Q)$ model using recent Pantheon+SHOES, Hubble, and DESI BAO observations.

Method, Experiments and Results

Theoretical Framework of $f(Q)$ Gravity

Symmetric teleparallel gravity provides an alternative geometrical description of gravitation in which the gravitational interaction is characterized by the non-metricity of spacetime rather than curvature or torsion. The action of $f(Q)$ gravity is given by [3,4]

$$S = \int \sqrt{-g}\left[\frac{f(Q)}{16\pi G} + \mathcal{L}_m\right] d^4x$$

where g denotes the determinant of the metric tensor and $\mathcal{L}_m$ is the matter Lagrangian density. The fundamental geometrical quantity is the non-metricity tensor $Q_{\gamma\mu\nu} = \nabla_\gamma g_{\mu\nu}$, whose traces are defined as $Q_\alpha = Q^{\mu}_{\alpha\mu}$, $\tilde{Q}_\alpha = Q^{\mu}_{\mu\alpha}$. To construct the gravitational dynamics, one introduces the superpotential tensor

$$4P^{\gamma}_{\mu\nu} = -Q^{\gamma}_{\mu\nu} + 2\, Q_{(\mu}{}^{\gamma}{}_{\nu)} + \left(Q^\gamma - \tilde{Q}^\gamma\right) g_{\mu\nu} - \delta^{\gamma}_{\mu} Q_\nu$$

Using which the non-metricity scalar is defined as $Q = Q_{\gamma\mu\nu}P^{\gamma\mu\nu}$.

Variation of the action yields the $f(Q)$ gravity field equations,

$$\frac{2}{\sqrt{-g}}\nabla_\gamma\left(\sqrt{-g}\, f_Q P^{\gamma}_{\mu\nu}\right) + \frac{1}{2} g_{\mu\nu} f + f_Q\left(P_{\nu\rho\sigma}Q_{\mu}^{\rho\sigma} - 2P_{\rho\sigma\mu}Q^{\rho\sigma}_{\nu}\right) = T_{\mu\nu}$$

Where $f_Q = \frac{df}{dQ}$ and $T_{\mu\nu} = -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{L}_m)}{\delta g^{\mu\nu}}$, the energy momentum tensor of the matter.

Cosmological Equations

To study the cosmological evolution, we consider a homogeneous, isotropic, and spatially flat FLRW background

$$ds^2 = -dt^2 + a^2(t)\delta_{ij}dx^i dx^j$$

For this geometry, the non-metricity scalar assumes the simple form $Q = 6H^2$, where $H = \frac{\dot{a}}{a}$ denotes the Hubble parameter.

Assuming the cosmic matter content can be described by a perfect fluid and adopting the decomposition $f(Q) = Q + F(Q)$, the modified Friedmann equations become

$$3H^2 = \rho + \frac{F}{2} - QF_Q$$

$$\left(2QF_{QQ} + F_Q + 1\right)\ \dot{H} + \frac{1}{4}\left(Q + 2QF_Q - F\right) = -2P$$

Where $F_Q = \frac{dF}{dQ}$ and $F_{QQ} = \frac{d^2F}{dQ^2}$. These equations determine the background dynamics of the Universe within the $f(Q)$ framework.

Power-Law Model

In the present work, we investigate the power-law realization of $f(Q)$ gravity [3,4, 6], given by $F(Q) = \alpha\left(\frac{Q}{Q_0}\right)^{\lambda}$, where α and λ are model parameters and $Q_0 = 6H_0^2$ denotes the current value of the non-metricity scalar. The model redices to the symmetric teleparallel equivalent of General Relativity when $\alpha = 0$. whereas the ΛCDM cosmology is recovered for λ=0. Introducing the normalized Hubble parameter $E(z) = \frac{H(z)}{H_0}$, the first Friedmann equation can be rewritten as

$$E^2 + \frac{\alpha}{Q_0}(2\lambda - 1)E^{2\lambda} = \Omega_m a^{-3}$$

Evaluating the above equation at the present epoch $(a_0 = 1)$ yields $\alpha = \frac{\Omega_m - 1}{2\lambda - 1}Q_0$. Substituting this result into the earlier expression we get,

$$E^2 + (\Omega_m - 1)E^{2\lambda} = \Omega_m a^{-3}$$

This equation is solved numerically to reconstruct the cosmic expansion history and to compute the theoretical observables employed in the MCMC analysis.

Observational Data and Statistical Analysis

To constrain the parameters of the power-law $f(Q)$ model, we employ Hubble, Pantheon+SHOES supernova compilation, and DESI baryon acoustic oscillation (BAO) data. The Hubble dataset consists of 32 cosmic chronometer measurements that provide direct estimates of the expansion rate of the Universe. The Pantheon+SHOES sample offers precise luminosity distance measurements from Type Ia supernovae, while the DESI dataset contributes 19 BAO measurements spanning different redshift ranges.

The model parameters are estimated using a Bayesian Markov Chain Monte Carlo (MCMC) analysis implemented through the Cobaya framework. The likelihood function is constructed from the total chi-square statistic. For the Pantheon+SHOES + Hubble combination, $\chi^2 = \chi_H^2 + \chi_{SN}^2$ whereas for the Pantheon+SHOES + Hubble + DESI combination, $\chi^2 = \chi_H^2 + \chi_{SN}^2 + \chi_{DESI}^2$.

The posterior distributions obtained from the MCMC analysis are used to determine the best-fit values and confidence intervals of the model parameters. The models are compared using the AIC and BIC criteria.

$$AIC = \chi_{min}^2 + 2k \quad , \qquad BIC = \chi_{min}^2 + k\ln N$$

where k, the number of free parameters and N denotes the total number of observational data points.

Result and Discussion: The observational constraints on the ΛCDM and power-law $f(Q)$ gravity models were obtained using the H(z)+Pantheon+SHOES and H(z)+Pantheon+SHOES+DESI datasets. The best-fit are listed in Table 2. For the combined dataset, both models provide similar estimates of the cosmological parameters. The power-law model yields a positive value of λ , indicating a mild deviation from the ΛCDM limit. The statistical comparison is presented in Table 1. According to the AIC criterion, both models provide comparable fits to the observational data. However, the BIC criterion moderately favors the simpler ΛCDM model due to its smaller number of free parameters. Figures (fig 1) and (fig2) show the posterior distributions and the reconstructed expansion history. The confidence contours exhibit the expected negative correlation between H_0 and Ω_{m_0}. Furthermore, the reconstructed Hubble expansion rate is in good agreement with the observational measurements, demonstrating the consistency of the power-law $f(Q)$ model with current cosmological data.

Models	χ^2	χ^2_{red}	AIC	ΔAIC	BIC	**ΔBIC**
ΛCDM	1450.397	0.8866	1458.397	0	1480.007	0
Power Law	1448.764	0.8861	1458.764	0.367	1458.577	5.769

Table 1. Comparison of Cosmological Models for combined Hubble+ Pantheon+SHOES+ DESI BAO data

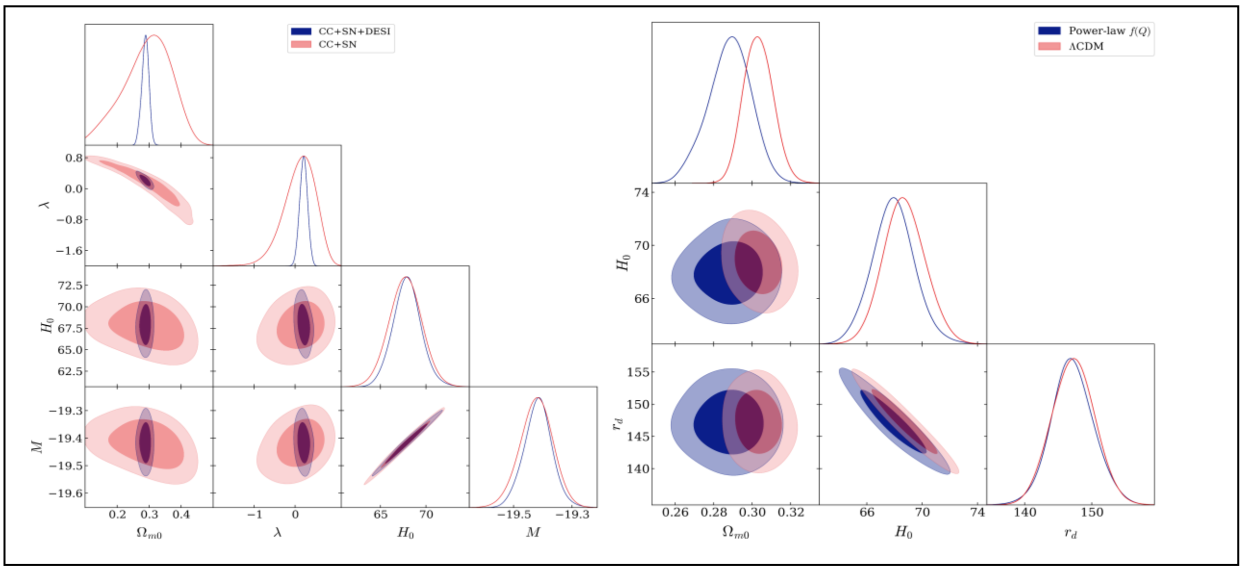

Figure 1: posterior distributions and 1σ - 2σ confidence contours and for the Power-law $f(Q)$ model (fig. a) and ΛCDM and Power-law $f(Q)$ models using the combined Pantheon+SHOES, $H(z)$, and DESI BAO datasets. (fig. b)

Models	Parameters	Priors	H(z)+Pantheon+SHOES	H(z)+Pantheon+SHOES+DESI BAO
ΛCDM	H_0	[60,80]	67.6188 ± 1.6428	68.9399 ± 1.4812
	Ω_{m_0}	[0.1,0.5]	0.3318 ± 0.0175	0.3040 ± 0.0076
	r_d	[120,180]	$-$	146.6007 ± 3.1295
	M	[-20,-18]	-19.4266 ± 0.0505	-19.3931 ± 0.0464
Power - Law	H_0	[60,80]	67.7951 ± 1.7719	68.2100 ± 1.5327
	Ω_{m_0}	[0.1,0.5]	0.3056 ± 0.0729	0.2894 ± 0.0111
	r_d	[120,180]	-	146.2409 ± 3.1407
	λ	[-2, 2]	0.1549 ± 0.3865	0.2187 ± 0.0951

M	[-20,-18]	-19.4187 ± 0.0559	-19.4063 ± 0.0476

Table 2. Best-Fit value $\pm 1\,\sigma$ of parameters from different models using H(z)+Pantheon+SHOES and H(z)+Pantheon+SHOES+DESI BAO datasets.

Conclusion: In this work, we constrained the power-law $f(Q)$ gravity model using cosmic chronometer, Pantheon+SHOES, and DESI BAO observations through MCMC analysis. The main conclusions of this study can be summarized as the power-law $f(Q)$ model successfully reproduces the observed expansion history and provides a fit comparable to ΛCDM. For the combined dataset, only a mild deviation from the ΛCDM limit is observed. Model comparison using AIC indicates nearly equivalent performance, while BIC moderately favors the simpler ΛCDM model

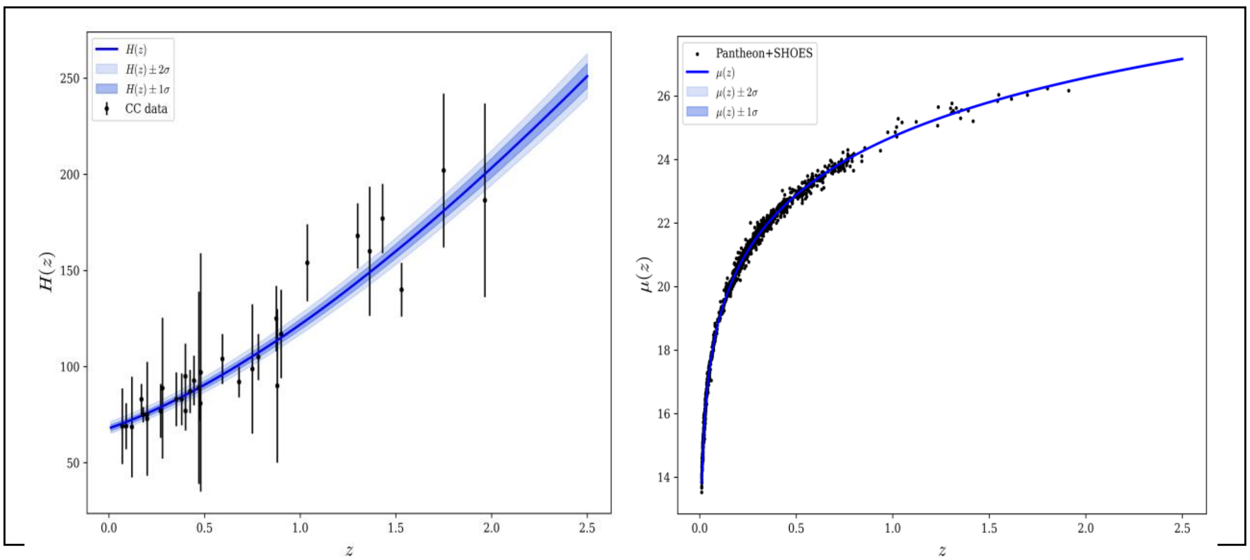

Figure2: Reconstructed $H(z)$ (fig c) and $\mu(z)$ (fig d) functions with 1σ and 2σ for the power law model.

References

1. A. G. Riess, et al. (1998). “Observational evidence from supernovae for an accelerating universe and a cosmological constant”. The astronomical journal, 116(3), 1009-1038. https://doi.org/10.1086/300499
2. S. Perlmutter, et al. (1999). “Measurements of Ω and Λ from 42 high-redshift super novae”. The Astrophysical Journal, 517(2), 565-586.https://doi.org/10.1086/307221
3. J. B. Jim´enez, L. Heisenberg, & Koivisto, T. (2018). “Coincident general relativity. ”Physical Review D, 98(4), 044048. https://doi.org/10.1103/PhysRevD.98.044048
4. J. B. Jim´enez et al. (2020). “Cosmology in f (Q) geometry”. Physical Review D, 101(10), 103507. https://doi.org/10.1103/PhysRevD.101.103507
5. R. Lazkoz, et al. (2019). “Observational constraints of f (Q) gravity”. Physical Review D, 100(10), 104027.https://doi.org/10.1103/PhysRevD.100.104027
6. D. Mhamdi, et al. (2024). “Constraints on power law and exponential models in f (Q) gravity”. Physics Letters B, 859, 139113. https://doi.org/10.1016/j.physletb.2024.139113

7. D. Brout, et al. (2022). “The Pantheon+ analysis: cosmological constraints.” The Astrophysical Journal, 938(2), 110.https://doi.org/10.3847/1538-4357/ac8e04
8. M. Abdul Karim, et al. (2025). “DESI DR2 results. II. Measurements of baryon acoustic oscillations and cosmological constraints.”
9. M. Moresco, et al. (2016). “A 6% measurement of the Hubble parameter at z $\sim$ 0.45: direct evidence of the epoch of cosmic re-acceleration”. Journal of Cosmology and Astroparticle Physics, 2016(05), 014-014. https://doi.org/10.1088/1475-7516/2016/05/014
10. J. Torrado, & A. Lewis, (2021). “Cobaya: Code for Bayesian Analysis of hierarchical physical models”. Journal of Cosmology and Astroparticle Physics, 2021(05), 057. https://doi.org/10.1088/1475-7516/2021/05/057