Quantum Electrodynamics Model for Quantum Plasma: Nonlinear Electromagnetic Wave Propagation Analysis

Nisha Singh Rtahore[1], Chandra Kumar Dixit[2]
[1]KCC Institute of Technology and Management, Greater Noida, Uttar Pradesh, India; [2]National Rehabilitation University, Lucknow, Uttar Pradesh, India.

Abstract: In this work, we study how dense quantum plasmas under extremely strong electromagnetic fields are affected by quantum electrodynamic (QED) corrections. In extreme plasma conditions, where quantum diffraction, electron degeneracy, relativistic effects, and vacuum polarisation all play significant roles in plasma behaviour, classical theories of plasma are insufficient. The quantum hydrodynamic equations and the modified Maxwell equations derived from the Heisenberg--Euler Lagrangian are used to create a theoretical model of QED plasma. Using perturbation techniques, the nonlinear propagation of electromagnetic waves is studied. The results show that the QED corrections have a considerable effect on the dispersion characteristics and nonlinear coupling coefficients. The results are useful for understanding nonlinear processes in magnetars, neutron stars, white dwarfs, and high intensity laser-plasma interaction studies.

Keywords: Quantum plasma; Quantum electrodynamics; Vacuum polarization; Nonlinear Schrödinger equation,

Introduction

One of the fastest-growing subfields of plasma science is quantum plasma physics, which has applications in high-energy-density physics, laser plasma interaction, semiconductor physics, and astrophysics. [3]–[7]. Quantum plasmas differ significantly from classical plasmas because they exhibit quantum mechanical processes as electron tunnelling, wave-particle duality, spin dynamics, and quantum statistical pressure [3]–[7].

In extremely dense plasma systems, the average interparticle distance is close to the thermal de Broglie wavelength. In such cases, classical thermal behaviour is subordinated to quantum processes. Furthermore, when ultra-intense electromagnetic fields are present, quantum electrodynamic (QED) effects become significant and modify the electromagnetic response of the plasma medium [8]–[10].

$$\mathscr{L} = \mathscr{L}_0 + (k/2)[(E^2 - c^2B^2)^2 + 7c^2(E\cdot B)^2]$$

where $\mathscr{L}_0$ is the classical electromagnetic Lagrangian and k is the QED correction coefficient. The Schwinger critical electric field is expressed as:

$$E_s = (m_e^2c^3)/ (e\hbar)$$

Nonlinear vacuum effects become very significant close to this field limit [1], [8], [9]. In this work, QED corrections and quantum diffraction effects are used to study the nonlinear propagation of electromagnetic waves in dense quantum plasma. The nonlinear Schrödinger equation paradigm is also used to investigate the instability of electromagnetic wave packets [2], [5], [7].

Theoretical Formulation

Consider an unmagnetized dense quantum plasma made up of stationary ions and electrons. The modified Maxwell equations in conjunction with the quantum hydrodynamic (QHD) equations control the plasma dynamics [3]–[5], [7].

The equation for continuity is provided by:

$$\partial n/\partial t + \nabla \cdot (nu) = 0$$

where n is the electron density and u is the fluid velocity. The momentum equation including the quantum Bohm potential is:

$$m_e (\partial u/\partial t + u\cdot\nabla u) = -eE - (1/n)\nabla P + (\hbar^2/2m_e)\nabla (\nabla^2\sqrt{n} / \sqrt{n})$$

where P is the electron degeneracy pressure and the last term represents quantum diffraction effects [3], [4], [7].

The modified electromagnetic wave equation including QED corrections becomes:

$$\nabla^2 E - (1/c^2)(\partial^2 E/\partial t^2) = \mu_0(\partial J/\partial t) + PQED$$

where PQED denotes the nonlinear vacuum polarization contribution [1], [2], [8]–[10].

Linear Dispersion Relation

To study the linear propagation characteristics, small perturbations of the form $\exp[i(kx - \omega t)]$ are assumed. Linearization of the governing equations yields the modified dispersion relation:

$$\omega^2 = \omega_p^2 + c^2k^2 + (\hbar^2k^4 / 4m_e^2) + \delta_{QED}k^2$$

where δQED represents the quantum electrodynamic correction parameter. The first term is associated with plasma oscillations, the second with the propagation of classical electromagnetic waves, the third with quantum diffraction, and the final one with vacuum polarisation effects [2], [4], [7]–[10]. Both quantum and QED corrections have a major impact on wave propagation, as the dispersion relation makes evident. While QED corrections become significant at very high electromagnetic field strengths, the quantum diffraction term becomes dominant at short wavelengths.

Nonlinear Schrodinger Equation

The reductive perturbation approach is used to study the nonlinear development of electromagnetic wave packets. The governing equations can be reduced to the nonlinear Schrödinger equation (NLSE) by using stretched coordinates and multiscale analysis.

$$i(\partial A/\partial \tau) + P(\partial^2 A/\partial \xi^2) + Q|A|^2 A = 0$$

where Q is the nonlinear coefficient, P is the group dispersion coefficient, and A is the slowly changing wave envelope amplitude. Wave number, QED correction parameters, and plasma density all affect the coefficients P and Q. The wave envelope's stability characteristics are determined by their signatures.

The instability of electromagnetic waves is analyzed by perturbing the steady-state solution of the NLSE. The instability condition is given by:

$$PQ > 0$$

The corresponding growth rate of instability is expressed as:

$$\Gamma = |P|K^2\sqrt{[(K_c^2/K^2) - 1]}$$

where K is the wave number and Kc is the critical wave number.

The analysis reveals that:

1. Quantum diffraction phenomena increase wave dispersion and stabilize the plasma system.
2. QED corrections enhance nonlinear interactions between electromagnetic waves and plasma particles
3. The instability bandwidth is greatly influenced by the plasma density and electromagnetic field strength.
4. Vacuum polarization increases the maximum growth rate of instability.

These findings suggest that QED events play a major role in energy localization and electromagnetic filamentation processes in dense plasma systems .

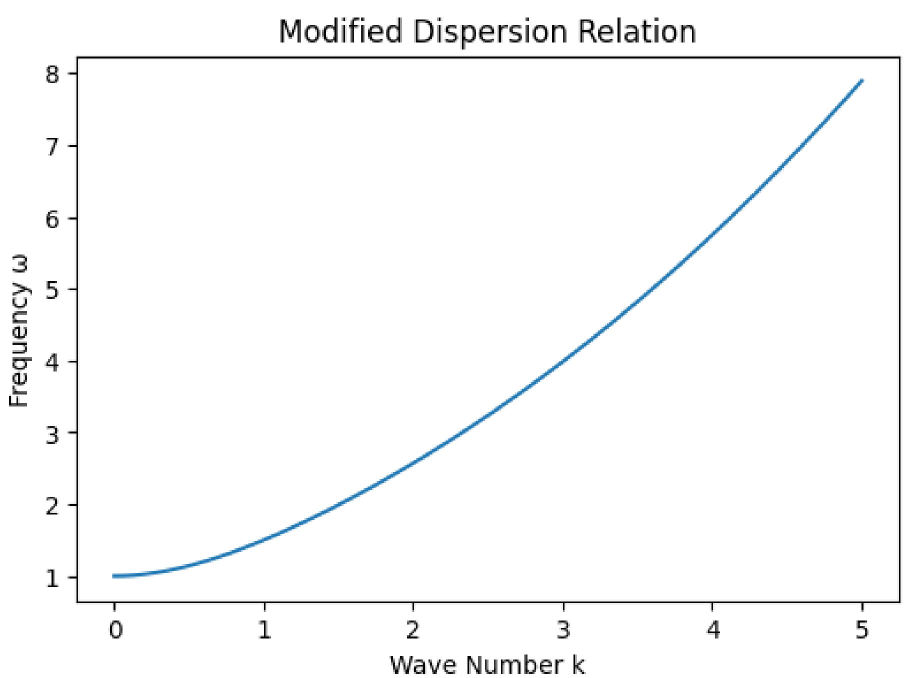

Figure: Modified Dispersion Relation

Discussion

The theoretical outcomes of this work are applicable to various laboratory and astrophysical plasma conditions. QED corrections are inevitable in neutron stars and magnetars because magnetic field intensities can surpass 10T. Similarly, electromagnetic intensities close to the Schwinger limit can be produced by next-generation laser facilities. Vacuum polarisation effects alter the nonlinear propagation characteristics of electromagnetic waves and the refractive properties of plasma. Localized structures such as envelope solitons and electromagnetic shocks may arise as a result of the combined action of quantum diffraction and QED nonlinearities.

Future research on relativistic spin plasmas, pair plasmas, and radiation response effects in ultra-intense laser plasma interaction may benefit from the current findings.

7. Conclusion

To study the propagation of nonlinear electromagnetic waves in dense quantum plasmas, a theoretical QED plasma model has been created. Modified wave propagation equations were developed by combining quantum diffraction effects via the Bohm potential with vacuum polarisation effects using the Heisenberg–Euler formalism. The analysis shows that nonlinear coupling, instability growth rates, and dispersion properties are significantly impacted by QED corrections. The findings offer important new information about how plasma behaves in relativistic and extreme quantum settings, which is pertinent to high-intensity laser operations and astrophysical compact objects. Future research could concentrate on relativistic spin dynamics, multidimensional wave propagation, and numerical simulation of nonlinear QED plasma systems.

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