

# Experimental and Numerical Investigation of Kerosene Jet Mixing in Supersonic Crossflow

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**Abstract:** Hydrocarbon-fuelled scramjets need fast fuel atomisation and fuel-air mixing in a very short residence time. Normally injected liquid kerosene in a supersonic crossflow is subjected to strong aerodynamic shear, shock interaction, deformation and rapid breakup, and direct measurement of the coupled gas-liquid field is difficult. This paper presents an integrated experimental-numerical system for studying kerosene-jet breakup and mixing in a Mach 2-3 supersonic crossflow. High-speed laser shadowgraphy and Schlieren imaging were combined with Detached-Eddy Simulation (DES) coupled to a Discrete Phase Model (DPM) and a Kelvin–Helmholtz/Rayleigh–Taylor (KH–RT) breakup formulation. At different Nozzle orifice diameters and various liquid-to-gas dynamic pressure ratios in the experiment, jet penetration, jet angle, empty-flow region, and shock wave structure were observed. The three features of the breakup zone are: a surface-wave-dominated region, a rapid-atomisation zone, and a downstream uniform-atomisation region. Increasing the ratio of liquid-to-gas dynamic pressure raised penetration by about 40% in the reported range, and reducing the nozzle diameter from 0.38 to 0.30 mm decreased the mean Sauter mean diameter by about 25%. The numerical model constructed the main penetration and shock structures, and supplied fields of velocity, pressure, density, vorticity and droplet size that could not be obtained experimentally. Build an experimental-computational basis for the interpretation of the penetration-atomisation trade-off in scramjet fuel injection through validation.

**Keywords:** scramjet; kerosene; supersonic crossflow; liquid-jet breakup; atomisation; DES; DPM; KH–RT model

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## 1. Introduction

Scramjet propulsion is a possible way to achieve prolonged high-speed air-breathing flight by carrying out combustion in a supersonic flow within a combustor. Scramjets are not rotating turbine-based propulsion systems; therefore, they lack a compressor and turbine, and their performance is highly dependent on the rapid injection, atomisation, mixing and ignition of fuel within a very short flow residence time [1-2]. Jet fuel is a good option for the hydrocarbon fuel system because of its high volumetric energy density and ease of storage and handling. However, it is not gaseous hydrogen; a liquid hydrocarbon needs to be injected, deformed, broken up and atomised, and then mixed to achieve a sufficiently uniform fuel-air distribution [3].

A supersonic crossflow has a large wall-normal momentum of the injected liquid and considerable streamwise momentum in the gas. Together, they generate a bow shock, separation region, shear layer, vortex structure and an expanding liquid column. Aerodynamic shear and interfacial instabilities generate ligaments and droplets, and penetration and atomisation are coupled phenomena [3-6].

The two necessary parameters of the current study are the nozzle orifice diameter,  $d$ , and the liquid-to-gas dynamic-pressure ratio,  $q$ . The latter is as follows:

$$q = \frac{\rho_l U_l^2}{\rho_g U_g^2} \quad (1)$$

where  $\rho_l$  and  $U_l$  are the liquid density and injection velocity, and  $\rho_g$  and  $U_g$  are the gas density and freestream velocity. Another index of the extent of air-breakup is the Weber number,

$$We = \frac{\rho_g U_{rel}^2 d}{\sigma} \quad (2)$$

$U_{rel}$  is the gas-liquid relative velocity and  $\sigma$  is the liquid-surface tension. The above amounts are used to find the connections between liquid momentum, aerodynamic force and interfacial surface tension.

Previous studies have identified the roles of nozzle geometry, momentum ratio, Weber number and freestream Mach number [4-8]. Optical diagnostics and numerical methods, such as LES, DES, DPM, VOF, and hybrid Eulerian-Lagrangian formulations, have enhanced the acquisition of jet and flow-field data [9-12]. However, there is still a persistent problem of a lack of close-knit experimental and numerical data for both penetration and breakup under scramjet-relevant conditions.

This paper studies normal kerosene injection in supersonic crossflow using a combination of optics and computation. The goal is not to provide all-encompassing divorce laws, but rather to build an all-round system for divorce and a path of reform. Sections 2–7 are the relevant works, contributions, methods, results, limitations and conclusions.

## 2. Related work and research gap

Liquid jets in gaseous crossflow have been studied for many years. Early supersonic experiments showed that jet penetration and lateral development are significantly affected by the relative momentum of liquid and gas [7, 8]. Later research distinguished between primary breakup of the coherent liquid column and secondary breakup of ligaments and droplets [5]. In the near field, the windward side of the column is subjected to a large aerodynamic load, and downstream structures have been modified by momentum transfer with the gas.

Nozzle Geometry and Size Affect Both Trajectory and Breakup. In general, a large-orifice has a higher mass-flow rate and momentum of the liquid, and a small characteristic length scale increases the surface-to-volume ratio, thus promoting deformation and breakup. Therefore, the high penetration and fine atomisation are not necessarily achieved by the same means [4-6].

Schlieren is used to observe density-gradient structures near shocks and shear layers, and shadowgraphy can be employed to obtain direct images of the liquid column and spray envelope. At supersonic speeds, the problems of motion blur, reduced contrast and line-of-sight ambiguity have not been solved [10-11].

Eulerian-Lagrangian methods can be used to describe the gas phase and dispersed droplets practically. DES/LES solve for unsteady gas motion, and DPM tracks droplets after breakup. KH-RT formulations are used for aerodynamics and acceleration-driven breakup; VOF/CLSVOF explicitly solves the main deformation at a higher computational cost [9], [12], [13].

Table 1 shows the position of the current study with respect to representative approaches.

Table 1. Comparison of the present study with representative previous approaches

| Study / approach              | Optical breakup | Shock structure | Numerical / sweep |
|-------------------------------|-----------------|-----------------|-------------------|
| Early penetration studies [7] | Limited         | Limited         | No / q, geometry  |
| Optical JICF studies [5],[10] | Yes             | Partial         | No / geometry     |
| LES/VOF studies [9],[12]      | Numerical       | Yes             | No / selected     |
| DES–DPM studies [13]          | Limited         | Yes             | Yes / selected    |
| Present study                 | Yes             | Yes             | Yes / d and q     |

This method has not been applied in research; therefore, neither the decomposition process nor verification data have been obtained from experiments or simulations. Combine both streams for research on Nozzle Diameter and Dynamic-Pressure Ratio.

### 3. Key Contribution

The first is an experimental basis for the coupling breakup and flow-field evolution. The three reasons are:

First, high-speed shadowgraphy shows the development of an initially coherent liquid column into surface-wave formation, rapid ligament fragmentation, and a downstream spray field. Therefore, the problem of division can be solved by dividing the area into several parts and then determining how to divide each part.

Second, high-speed Schlieren imaging can resolve the associated bow-shock and separation-shock topology. Another measure of the decrease in airflow due to a liquid column is the shock angle.

Third, the DES-DPM framework can be used to add more information on the distribution of velocity, pressure, density, vorticity, penetration and droplet-size in a KH-RT breakup formulation. A Mach 2 reference case was selected for the calibration of the numerical framework, and grid independence was confirmed at 4.5 million cells. Calibration of the breakup-time parameter resulted in  $B_1 = 5$  for the reference case.

## 4. Methods and experimental–numerical framework

### 4.1 Experimental configuration

A pulse supersonic wind-tunnel with optical access to the injection region was used in the experiment. The reported operating conditions of the main experimental matrix are as follows: Freestream Mach number = 3, Static temperature  $\approx 300$  K, Total pressure = 0.8 MPa, and Injection angle =  $90^\circ$  to the wall. Liquid RP-3 aviation kerosene was introduced via circular laser-drilled orifices of 0.30, 0.38, 0.50 and 0.80 mm in diameter. Based on the orifice size, the calculated liquid-to-gas dynamic-pressure ratio  $q$  is about 3–22.

Combine pulsed laser shadowgraphy and high-speed schlieren for the imaging system. Nanosecond-scale laser pulses reduced motion blur; Schlieren imaging was used for compressible flow. Both systems were synchronous to the injection event.

Figure 1 is the three-zone interpretation extracted from the shadowgraph observation. Close to the nozzle, in the surface-wave-dominated area, a separate liquid column and developing interface disturbances are generated. Downstream of the rapid-atomisation Area, there is serious ligament

fragmentation and a dense spray. In the uniform-atomization region, the distance between the droplets in space is relatively large, and the gas and liquid velocities are close to each other.

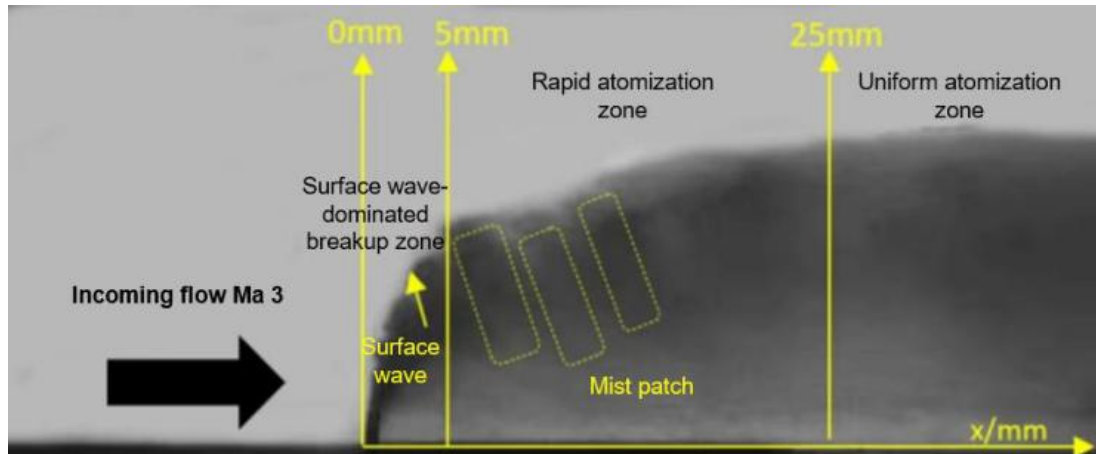


Fig. 1. Observed breakup and atomisation sequence in the supersonic crossflow: surface-wave-dominated, rapid-atomisation, and downstream uniform-atomisation zones.

#### 4.2 Image processing and measured quantities

The instantaneous jet angle  $\theta(t)$  is the location of the local jet-column centreline. Columns are naturally unstable and will move slightly on their own in the absence of other forces. Therefore, the mean angle of a representative point in the stable injection window was used.

Based on the determined spray boundary, penetration was calculated as a function of the streamwise location downstream of the nozzle. There was also an empty-flow area in the optical images. The area is the region between the nozzle and the first clear spray structure; although very fine droplets may still be present in this area, they will be below the segmentation threshold.

The shock-wave angle  $\alpha$  was taken in the far-field region of the bow shock, and it is almost constant and oblique there. Systematic errors were mainly in the spatial and pressure calibrations, and the random errors were due to repeatability and image segmentation.

#### 4.3 Numerical model

The size of the 80mm x 30mm x 30mm flow passage for calculation and a wall-normal liquid injection orifice at the bottom wall were selected. A compressible DES formulation based on a modified Spalart-Allmaras closure was employed to solve the gas-phase problem. The Liquid Phase was modelled by a Lagrangian DPM. Momentum transfer of gas-particles has been added, and under the DPM method, droplet collision, coalescence and breakup occur.

The gas-phase continuity equation is

$$\frac{\partial(\alpha_g \rho_g)}{\partial t} + \frac{\partial(\alpha_g \rho_g u_{gj})}{\partial x_j} = 0 \quad (3)$$

The corresponding momentum equation is

$$\frac{\partial(\alpha_g \rho_g u_{gi})}{\partial t} + \frac{\partial(\alpha_g \rho_g u_{gi} u_{gj})}{\partial x_j} = -\alpha_g \frac{\partial p_g}{\partial x_i} + \frac{\partial \tau_{ij}}{\partial x_j} + f_{gpi} \quad (4)$$

where  $\alpha_g$  is gas volume fraction,  $\rho_g$  is gas density,  $u_g$  is gas velocity,  $p_g$  is pressure,  $\tau_{ij}$  is viscous stress tensor, and  $f_{gpi}$  is gas-particle interaction term.

The two-phase model has Kelvin-Helmholtz and Rayleigh-Taylor instabilities. The growth rate and characteristic wavelength in the KH part are affected by the gas Weber number and liquid properties. The characteristic breakup time was shown in the model as

$$\tau_{KH} = \frac{3.726 B_1 a}{\Lambda_{KH} \Omega_{KH}} \quad (5)$$

'a' is the parent-droplet radius, ' $\Lambda_{KH}$ ' is the most amplified wavelength, ' $\Omega_{KH}$ ' is the corresponding growth rate, and ' $B_1$ ' is the calibrated breakup time parameter. RT is calculated according to the growth rate, wavelength and breakup-time expressions given in the model formulation of this thesis.

Mach 2 was chosen, and  $d = 0.30, 0.50$ , and  $0.70$  mm were selected as the spacing values for  $q$ . As the experimental and numerical matrices are different, only the corresponding cases can be quantitatively compared.

#### 4.4 Numerical verification and validation

Reference simulation at Mach 2,  $d = 0.5$  mm and  $q = 4.8$  was used for breakup model calibration. As shown in  $B_1$ , a value less than 5 is too low, and a larger value is too high.  $B_1 = 5$  was thus chosen.

Grid-independence analysis used 2.5, 4.5 and 6.5 million cells. A baseline of a 4.5-million-cell grid was chosen because its penetration trajectory was about 3.7 per cent different from that of the experimental curve, and the higher computational cost of a 6.5-million-cell grid was avoided. Then, the above-mentioned validated simulations were used for parametric analysis.

### 5. Results and discussion

#### 5.1 Breakup topology

Shadowgraph observations show a slow decline in the injected kerosene. Soon after the nozzle, there has been no significant displacement of the liquid column, and a surface wave has formed on the windward side. With an increase in the distance downstream, both the amplitude and the displacement from the liquid core of the ligaments are larger. A high-density droplet-field structure is used for the arrangement. The Spray will be further spread downstream.

First of all, a simple and easy-to-understand diagram of a jet is presented below. The near-nozzle liquid column still has a relatively large amount of the initial injection momentum. When the crossflow reaches the column, aerodynamic momentum is transferred from the gas to the liquid, and as a result, the column is pushed downstream. Interfacial disturbances lead to both the growth and decay of liquid-structure characteristic size simultaneously. Therefore, the two processes of penetration and atomisation occur concurrently rather than one after the other.

The observed windward-side stripping is consistent with the vorticity field in Figure 2.10. The maximum gas-phase vorticity occurs in the near-field region of interaction between the crossflow and the liquid column. Therefore, this Area is relatively active in terms of interfacial deformation and droplet removal.

#### 5.2 Effect of dynamic-pressure ratio

Maintain the nozzle diameter and increase  $q$  to widen the jet angle and extend the range of penetration. For  $d = 0.38$  and  $d = 0.30$  mm, when  $q$  was increased from 5 to 11, the mean jet angle also increased by about 5 degrees. Related Penetrations have also risen. According to the reported

test data, the thesis abstract and conclusion show that with an increase in  $q$ , penetration has risen by about 40%.

For a constant crossflow, an increase in  $q$  will result in a higher momentum of the liquid injection. Thus, the column has a wall-normal momentum further downstream and is then redirected for transverse penetration.

The Size of the optical empty-flow region is proportional to  $q$ . According to the penetration results, an increase in the injection momentum will shift the centre of the break-up area downstream. A dead-flow metric should be used for the auxiliary optical indicator, and it must not be used to measure the total liquid volume directly, otherwise, droplets smaller than the segmentation threshold may still be present in the apparent empty area.

The Effect of  $q$  on Atomisation is relatively small. The numerical results for  $d = 0.5$  mm and  $q = 2, 3.3$ , and  $5.8$  show that a smaller  $q$  results in stronger aerodynamic interaction because, under the specified matched conditions, the relative gas-liquid velocity is higher. Therefore, the Weber number is relatively large, and the breakup will be promoted. The low- $q$  case has a smaller area of large droplets and a more uniform SMD distribution downstream. Therefore, increasing  $q$  improves coverage but not necessarily the fineness of atomisation.

### **5.3 Effect of nozzle orifice diameter**

Nozzle Diameter will change the liquid column to a certain extent. Fix  $q$  and increase  $d$ ; both the liquid mass flow rate and its momentum will rise. According to the experiment, the mean jet angle of the larger orifice is higher. The numerical results show that the penetration and the range of gas-phase blockage are also larger for a larger  $d$ .

A large opening is a relatively high-pressure region and a strong bow shock, so aerodynamic blockage will increase.

However, the same increase in diameter will reduce the fineness of atomisation. According to the distribution of numerical SMDs, the larger orifice has a relatively higher proportion of large droplets, particularly on the windward side and in the middle of the spray. Small-orifice nozzles are relatively easy to deform, and a fine-droplet stream is emitted downstream. Based on the characteristic surface-to-volume ratio, a smaller  $d$  increases the available liquid surface per unit volume of liquid and enhances the efficiency of aerodynamic momentum transfer.

According to the quantitative results presented in the abstract of this thesis, reducing the nozzle diameter from 0.38 mm to 0.30 mm lowered the mean SMD by about 25%. Therefore, the result above is used here, as it is directly presented in the consolidated thesis results. In general, due to the limitations of experimental matrices and optical windows, the relationship between diameter and penetration can only be shown qualitatively, as a single universal quantitative scaling of the diameter for all cases is not supported.

### **5.4 Shock-wave structure**

According to the Schlieren images, the shock system upstream of the liquid column is entry-shaped. It has a bow shock wave, a separation zone and a separation shock wave. The three characteristic areas of the bow shock are shown below. The column is almost circular near the wall; therefore, the local displacement will be relatively large. Move the column further downwind of the wall to reduce the effective obstruction. The shock wave in the outer region is almost a constant oblique wave.

Figure 2 is a typical structure. Near the wall, this area is close to a sudden-expansion wave, and further downstream, a less abrupt oblique-shock wave occurs after the gas-liquid momentum transfer to the liquid column and droplets.

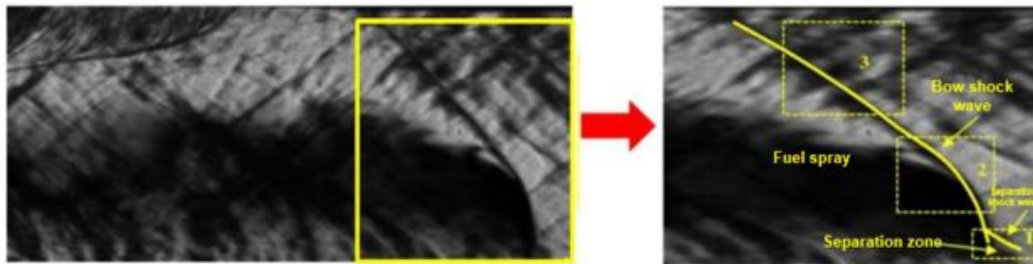


Fig. 2. Schlieren observation of the bow-shock, separation-shock, and separation-zone structure for Mach 2,  $d = 0.3 \text{ mm}$ ,  $q = 17$ .

The mean shock angle in the far-field region increased from  $39.86^\circ$  to  $40.85^\circ$  with an increase in  $q$  from 17 to 22 for  $d = 0.3 \text{ mm}$ . At  $q = 17$ , changing  $d$  from  $0.3 \text{ mm}$  to  $0.5 \text{ mm}$  results in the mean angles of  $39.87^\circ$  and  $41.66^\circ$ , respectively. As shown in the modifications above, both the injection momentum and the nozzle diameter affect aerodynamic blockage. As shown in the notes of the thesis, only the two  $q$  values for this specific shock-angle comparison were available, and the  $0.8 \text{ mm}$  case at  $q = 17$  exceeded the optical window. Thus, although it is convenient to obtain such data, they should not be assumed to be generally applicable.

### 5.5 Numerical penetration and droplet fields

The direction and magnitude of the trajectory of the numerical penetration are in line with the published empirical correlation. The Magnitudes of Correlation are the same.

$$\frac{y}{d} = A \left(\frac{x}{d}\right)^B q^C \quad (6)$$

A, B, and C are the regression coefficients of the published correlation coefficient. The range of the simulations is generally the same; however, there are local differences due to the resolved unsteady breakup.

Figure 3 is this comparison. The agreement will be used to run the calibrated DES-DPM framework in the following parametric studies.

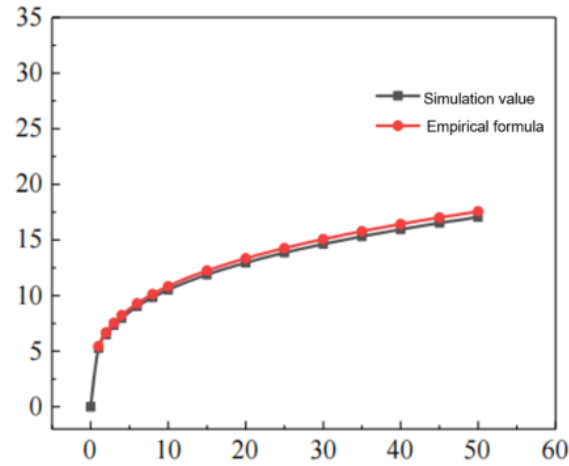


Fig. 3. Simulated particle penetration trajectory compared with the published empirical correlation.

As shown in the numerical droplet field, the droplets are relatively larger near the windward side and smaller downstream and close to the wall; this is in line with aerodynamic stripping followed by secondary breakup.

The distribution of droplet velocity is also subject to the same momentum-transfer mechanism; therefore, the initial wall-normal component is converted into a streamwise velocity, and smaller droplets are more sensitive to changes in the gas flow.

### 5.6 Dynamic-pressure ratio and droplet-size distribution

Figure 4 shows the computed droplet-size distributions for  $d = 0.5$  mm at  $q = 2, 3.3$ , and  $5.8$ . The larger-droplet population is concentrated in the near-wake area close to the nozzle. Further down the road, the jets will be spread out.

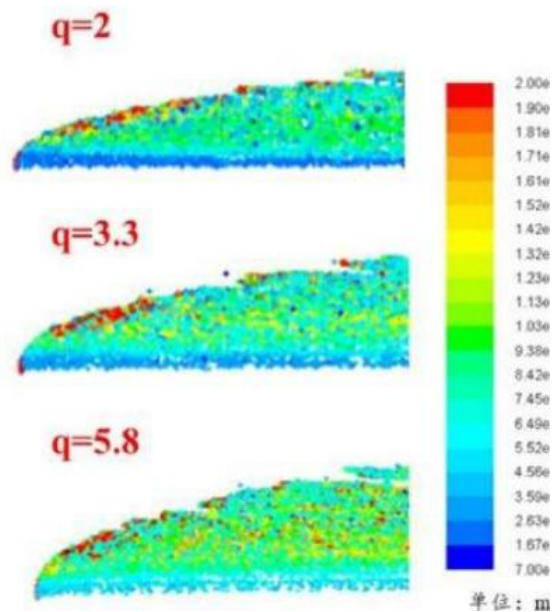


Fig. 4. Computed droplet-size distribution for  $d = 0.5$  mm at  $q = 2, 3.3$ , and  $5.8$ .

In the lower- $Q$  case, the area of the large-droplet concentration is smaller, and the downstream SMD field is more uniform. Given the conditions of the numerical sweep, a smaller  $q$  is associated with a

larger relative velocity and thus with a larger Weber number, as shown in Eq. (2). An increase in Aerodynamic Force will increase the instability of the KH-RT interface and lead to breakup. Therefore, the condition that leads to a lower penetration will also have finer atomisation.

Penetration and Atomisation are not the same measures of mixing quality. A good penetration condition will lead to a thick column; otherwise, due to severe aerodynamic breakup, there will be fewer droplets and poor penetration. Therefore, the conditions of the injector need to be selected to balance the transverse fuel delivery and atomisation rate.

## **6. Limitations and engineering implications**

Some deficiencies limit the range of the current results. First, the experimental and numerical baselines were not identical. Mach 3 was used for the main experiment, and the numerical parameter study was carried out at Mach 2. Therefore, only the corresponding cases could be directly counted.

Secondly, the Optical Observation Window has some high-penetration defects. Notably, no combinations of larger orifices and higher  $q$  were found in the whole field of view. Third, only based on the two  $q$  values per diameter, the shock-angle dependence of  $q$  was studied, and a general exponent could not be determined reliably. Fourthly, a numerical model of a non-reacting, essentially isothermal flow was established, and the full thermo-chemical conditions of a fired scramjet combustor were not taken into account.

The Deficiencies Limit Expansion. Next, the experimental and numerical matrices will be aligned in the future; then, the study will be extended to a high-total-temperature region where evaporation and variable-fuel properties are pronounced; finally, reacting chemistry will be added.

Therefore, the engineering results of the current experiments are a trade-off among the parameters rather than a single optimum. A large-orifice, high- $q$  is likely to penetrate better and produce a stronger aerodynamic obstruction of the liquid column. A small opening and stronger air-breakup promote fine-droplet generation. An authenticated experiment-numerical framework has been constructed to study the trade-off in the absence of other factors, such as injection angle, multiple-orifice arrangement, cavities, ramps or reacting flow.

## **7. Conclusion**

An integrated experimental-numerical study of liquid kerosene injection in a supersonic crossflow has been conducted. High-speed laser shadowgraphy and Schlieren imaging were combined with the DES-DPM framework to add the KH-RT breakup model.

Based on the results of the experiment, three types of breakup have been identified in the experiment: a surface-wave-dominated region near the nozzle, a rapid-atomisation region with a large number of ligaments and droplets, and a downstream region where the spray is relatively uniform. The three regions of the shock-wave field are the bow shock, the separation region, and the separation shock.

Dynamic Pressure Ratio of Liquid-Gas Affects Penetration. Increasing  $q$  increased the jet angle and penetration, and at all the tested values, the penetration was about 40% larger. A small  $q$  increased the angle of aerodynamic breakup and improved the distribution of downstream droplets in the numerical sweep. A nozzle with a larger diameter can achieve thicker, deeper-penetration atomisation; at the same time, it is also suitable for near-field blocking of a large-orifice nozzle, and a smaller-orifice nozzle will result in finer atomisation. Based on the above results, a reduction of 0.08 mm in the degree of reduction is associated with an approximately 25% decrease in mean SMD.

A calibrated numerical framework has reconstructed the main penetration and shock structures, and gas-phase and droplet data have been incorporated. Based on the above results, to achieve effective fuel injection for a scramjet, both penetration and atomisation are required; neither can be used alone. Therefore, this study provides a basis for further optimisation of injectors and combustors.

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