Trajectory Tracking of a 3-DOF Parallel Manipulator Using Optimally Tuned LQR-PID Controller

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Abstract: This paper presents a comparative analysis of trajectory tracking control for a three-degree-of-freedom (3-DOF) parallel manipulator utilizing an optimally configured Linear Quadratic Regulator-Proportional Integral Derivative (LQR-PID) controller. The inherent nonlinearity and coupled dynamics of parallel manipulators present significant challenges in controller parameter selection. To identify suitable controller parameters, three optimization approaches—Genetic Algorithm (GA), Particle Swarm Optimization (PSO), and Grey Wolf Optimizer (GWO)—are implemented. The LQR weighting parameters and PID gains are concurrently optimized through minimization of a composite objective function incorporating Sum Squared Error (SSE), Integral Squared Error (ISE), and Integral Absolute Error (IAE). The optimized controllers undergo evaluation across multiple reference trajectories of the manipulator. Comparative simulation findings indicate that all three optimization techniques enhance tracking performance; notably, GWO delivers the most precise and consistent trajectory tracking among the examined approaches. The GWO-optimized controller demonstrates reduced tracking error, diminished integral error, and superior control-torque characteristics relative to GA- and PSO-optimized controllers. These findings validate the efficacy of GWO for optimal configuration of LQR-PID controllers in precision trajectory tracking applications for parallel robotic manipulators.

Keywords— 3-DOF parallel manipulator, trajectory tracking, LQR, PID, Genetic Algorithm, Particle Swarm Optimization, Grey Wolf Optimizer, optimal control, robotic control.

1. INTRODUCTION

Parallel manipulators have garnered substantial interest due to their exceptional stiffness, minimal moving mass, superior payload capacity, and capability for high-precision motion [1], [2]. These attributes render them well-suited for manufacturing, medical robotics, aerospace systems, and pick-and-place applications. However, controlling a parallel manipulator presents significant challenges [3], [4] because its dynamics exhibit nonlinearity and strong coupling. Fluctuations in configuration, actuator properties, friction, payload, and environmental disturbances can substantially impact trajectory tracking performance [5]–[8]. Consequently, an effective controller and methodical parameter-tuning methodology are essential. PID controllers are extensively employed due to their straightforward architecture and ease of practical application [9], [10]. Nevertheless, manual determination of Kp, Ki, and K_d parameters proves problematic for nonlinear multivariable robotic systems. LQR delivers an optimal state-feedback control strategy by reducing a quadratic performance metric. Its effectiveness, however, relies significantly on how the state and control weighting matrices are selected.

A hybrid LQR-PID controller can integrate the strengths of state feedback with conventional error-based control approaches. Yet, determining both LQR weighting parameters and PID gains concurrently

constitutes an optimization challenge [11]. Population-based optimization algorithms offer a viable solution for addressing this problem [12]–[15]. GA explores through evolutionary mechanisms, PSO leverages coordinated particle dynamics [12], [13], and GWO applies the hunting strategy and hierarchical structure of grey wolves [14]. This research applies GA, PSO, and GWO to optimize the LQR-PID controller for a 3-DOF parallel manipulator [16]-[20]. The primary goal is to evaluate and compare these three optimization methodologies using consistent performance metrics.

The principal contributions of this investigation encompass:

1. Formulation of an LQR-PID-based trajectory tracking methodology applicable to a three-degree-of-freedom parallel manipulator.
2. Concurrent parametric optimization of LQR and PID controllers utilizing genetic algorithms, particle swarm optimization, and grey wolf optimization techniques.
3. Quantitative performance evaluation employing sum of squared errors, integral of squared error, and integral of absolute error metrics.
4. Comparative analysis of trajectory tracking fidelity and control torque characteristics.
5. Determination of the optimization algorithm yielding superior tracking accuracy and response smoothness.

2. THREE DEGREE OF FREEDOM (3-DOF) PARALLEL MANIPULATOR MODEL

This scholarly investigation focuses on the Maryland parallel manipulator, a spatial parallel mechanism characterized by three degrees of freedom (3-DOF), as illustrated in Figure 1. The apparatus comprises a mobile platform, a stationary platform, and three identical kinematic limbs. Each limb establishes a connection between the fixed and mobile platforms, thereby constituting a closed kinematic chain. The structural composition of each limb includes an active link and a passive link. The passive components are constructed as four-bar parallelogram mechanisms, which are articulated with the input links and the mobile platform. All constituent elements—including the platforms (both stationary and mobile), links, and joints—are presumed to exhibit rigid body characteristics, precluding any deformation under operational conditions. The interconnections among the platform, input links, and passive links are facilitated through parallel revolute joints for each limb. Consequently, the manipulator exhibits precisely three degrees of freedom.

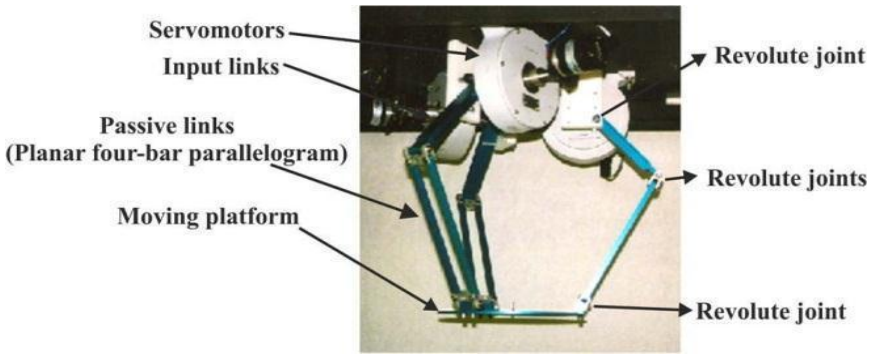


Figure.1. Maryland manipulator

3. PROPOSED OPTIMALLY TUNED LQR-PID CONTROLLER

A. LQR Controller

The Linear Quadratic Regulator (LQR) controller seeks to minimize the quadratic performance index J(u)= $\int_0^\infty [\mathrm{x^T(t)Qx(t)} + \mathrm{u^T(t)Ru(t)}]\mathrm{dt}$

wherein Q represents the state weighting matrix and R denotes the control-input weighting matrix.

The optimal feedback control law is expressed as: u(t)= $-\mathrm{R^{-1}B^TPx(t)} = -\mathrm{F}x(t)$

where, F= $-\mathrm{R^{-1}B^TP}$ and the P matrix is determined through the solution of the algebraic Riccati equation.

$\mathrm{A^TP + PA - PBR^{-1}B^TP + Q = 0}$

The effectiveness of the LQR controller is substantially influenced by the appropriate selection of weighting matrices Q and R.

B. PID Controller

The PID controller generates a corrective control signal predicated upon the tracking error:

$e(t) = q_d(t) - q_a$(t)

where, q_d = desired active joint angle expressed in radians

q_a = actual active joint angle expressed in radians

The PID control law is formulated as:

$$u_{PID}(t) = K_p e(t) + K_i \int_0^t e(\tau)d\tau + K_d \frac{de(t)}{dt}$$

The proposed hybrid controller integrates LQR state feedback with PID error compensation. The parameters optimized through metaheuristic algorithms are denoted by-

$X = [Q_1, Q_2, Q_3, R, K_p, K_i, K_d]$, Consequently, the optimization problem encompasses seven controller parameters.

4. OPTIMIZATION METHODOLOGY

A. Objective Function

The tracking performance is assessed utilizing sum square error (SSE), integral square error (ISE), and integral absolute error (IAE).

$\mathrm{SSE} = \frac{1}{\mathrm{N}}\sum_{\mathrm{i=1}}^{\mathrm{N}}(\mathrm{e_i})^2$, $\mathrm{ISE} = \int_0^\infty [\mathrm{e(t)}]^2\mathrm{dt}$ $\mathrm{IAE} = \int_0^\infty [\mathrm{e(t)}]\mathrm{dt}$

A composite objective function is established as: $\mathbf{J}(\mathrm{k_i, k_p, k_d}) = \mathrm{w_1 SSE + w_2 ISE + w_3 IAE}$

For the comparative investigation: $\mathrm{J}(\mathrm{k_i, k_p, k_d}) = 1.\mathrm{SSE} + 2.\mathrm{ISE} + 1.\mathrm{IAE}$

The optimization problem is thus formulated as; $\mathrm{X^* = \min J(k_i, k_p, k_d)}$

The identical objective function and controller architecture are employed across GA, PSO, and GWO to facilitate equitable comparison.

B. Genetic Algorithm

GA conceptualizes the controller parameters as a chromosome. A population of prospective solutions is initialized and evaluated. Subsequently, selection, crossover, and mutation operations are applied iteratively. The optimization procedure is presented in fig 2.

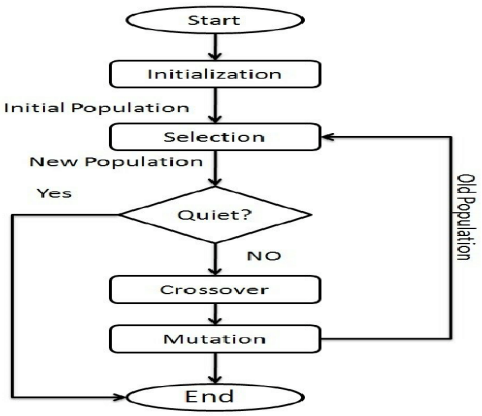


Figure 2. Algorithms steps of GA

C. Particle Swarm Optimization

Within the Particle Swarm Optimization (PSO) framework, each particle constitutes a candidate controller parameter vector. The particle velocity and position are updated according to the following formulations:

$v = [v_{i1}, v_{i2}, v_{i3}, v_{in}]$ & $y = [y_{i1}, y_{i2}, y_{i3}, y_{in}]$

where, i=1,2,3……,n

$v_i^{t+1} = wv_i^t + c_1 r_1 (P_{i,best}^t - y_i^t) + c_2 r_2 (G_i^t - y_i^t)$ $y_i^{t+1} = y_i^t + v_i^{t+1}$

In these expressions, $\mathbf{v_i^{t+1}}$ represents the current velocity, v_i^t denotes previous velocity, y_i^{t+1} indicates present position, y_i^t indicates previous position, $P_{i,best}^t$ denotes particle best position, G_i^t denotes global best solution, and t is defined as the iteration number (t=100).

D. Grey Wolf Optimizer

The Grey Wolf Optimizer (GWO) algorithm emulates the hierarchical social structure and cooperative hunting mechanisms observed in grey wolf populations. The three superior solutions are designated as alpha, beta, and delta wolves, respectively.

$\overrightarrow{W_1} = |\overrightarrow{W_\alpha}(t) - \overrightarrow{M_1}.\overrightarrow{Q_\alpha}|, \overrightarrow{W_2} = |\overrightarrow{W_\beta}(t) - \overrightarrow{M_2}.\overrightarrow{Q_\beta}|$ & $\overrightarrow{W_3} = |\overrightarrow{W_\delta}(t) - \overrightarrow{M_3}.\overrightarrow{Q_\delta}|$

The vectors $\overrightarrow{Q_\alpha}$, $\overrightarrow{Q_\beta}$ and $\overrightarrow{Q_\delta}$ are derived from the preceding equations as follows:

$\overrightarrow{Q_\alpha} = |\overrightarrow{S_1}.\overrightarrow{W_\alpha} - \overrightarrow{W}|$, $\overrightarrow{Q_\beta} = |\overrightarrow{S_2}.\overrightarrow{W_\beta} - \overrightarrow{W}|$ & $\overrightarrow{Q_\delta} = |\overrightarrow{S_3}.\overrightarrow{W_\delta} - \overrightarrow{W}|$

Utilizing the aforementioned equations, the updated solution for the subsequent iteration is obtained through: $\overrightarrow{W}(t+1) = \frac{\overrightarrow{W_1} + \overrightarrow{W_2} + \overrightarrow{W_3}}{3}$

Should the maximum iteration threshold or optimal solution criterion remain unattained, a new candidate solution is generated. Subsequently, fitness value evaluation is performed, and based on the comparative fitness assessment between the current and previous solutions, iterative refinements are systematically applied on a continuous basis.

Finally, the composite matrix Q is defined as: Q = diag [Q_1, Q_2, Q_3] = diag [$\overrightarrow{Q_\alpha}$, $\overrightarrow{Q_\beta}$, $\overrightarrow{Q_\delta}$]

5. SIMULATION AND RESULTS

The controller undergoes evaluation across three sequential reference trajectories. These trajectories incorporate both vertical and horizontal motion components to assess controller performance under diverse operational scenarios. The desired platform trajectory is converted into joint-space reference trajectories through the application of inverse kinematics. Subsequently, the actual joint trajectories are derived from the dynamic simulation model. The GA, PSO, and GWO algorithms are configured with identical population sizes, maximum iteration counts, parameter bounds, and objective functions. The optimized gain parameters for the LQR-PID controller are presented in Table 1 below.

TABLE 1: OPTIMIZED LQR-PID PARAMETERS

Parameter	GA	PSO	GWO
Q_1	167.87	212.06	369.32
Q_2	214.56	162.27	181.19
Q_3	142.63	218.83	187.63
R	0.35	0.10	0.08
K_i	21.749	38.213	67.944
K_p	37.685	61.573	96.117
K_d	19.397	38.034	43.003

The optimized parameter sets reveal that each optimization algorithm achieves convergence toward a distinct combination of LQR and PID parameters. The GWO algorithm yields a parameter combination that produces the minimal tracking error.

A. Tracking Performance

All three optimized controllers successfully track the reference trajectory; however, the Grey Wolf Optimizer (GWO)-based controller demonstrates the most precise alignment with the desired trajectory, exhibiting the minimal error magnitude and the most attenuated transient response characteristics. The quantitative performance metrics are presented in Table 2.

TABLE 2: INPUT-LINK TRACKING PERFORMANCE

Method	SSE	ISE	IAE	Objective J
GA	1.395	0.0110	0.1120	1.5180
PSO	0.895	0.0068	0.0672	0.9690
GWO	**0.512**	**0.0030**	**0.0213**	**0.5363**

The GWO controller attains the minimum values across all four performance indices. In comparison with the Genetic Algorithm (GA), the GWO controller achieves reductions in Sum of Squared Errors (SSE) of approximately 63.3%, Integral of Squared Error (ISE) of 72.7%, Integral of Absolute Error (IAE) of 81.0%, and the composite objective function of approximately 64.7%. When compared with Particle Swarm

Optimization (PSO), the corresponding reductions are approximately 42.8%, 55.9%, 68.3%, and 44.7%, respectively. These findings demonstrate that the GWO algorithm delivers substantial enhancements in both instantaneous and cumulative trajectory tracking error mitigation.

B. Passive-Link Tracking

The efficacy of the passive linkages is assessed concurrently, as precise input-link monitoring alone does not ensure accurate kinematic behavior throughout the entire parallel mechanism.

TABLE 3: TRACKING PERFORMANCE OF PASSIVE LINKS

Link	Method	SSE	ISE	IAE
Passive Link 1	GA	3.4355	0.0288	0.1652
	PSO	1.4091	0.0085	0.0429
	GWO	**1.1325**	**0.0058**	**0.0143**
Passive Link 2	GA	5.9796	0.0475	0.1756
	PSO	2.7834	0.0155	0.0463
	GWO	**2.4761**	**0.0124**	**0.0167**

The Grey Wolf Optimizer algorithm consistently yields the minimum error magnitudes. These findings demonstrate that the enhancements achieved through the optimization process propagate throughout the complete mechanism architecture rather than remaining localized to the actuated input link.

C. Control Torque Comparison

The control torque produced by the three optimized control systems is scrutinized to determine the continuity and feasibility of the control methodology.

TABLE 4: CONTROL-TORQUE PERFORMANCE

Method	SSE	ISE	IAE
GA	1.2261	0.0103	0.1020
PSO	0.7915	0.0059	0.0616
GWO	**0.4656**	**0.0027**	**0.0196**

The Grey Wolf Optimizer controller demonstrates the lowest torque tracking deviations. This observation suggests that enhanced positional tracking performance is accompanied by more refined and judicious control action, rather than necessitating excessive actuator demands.

6. CONCLUSION

An investigation into the comparative performance of trajectory-tracking methodologies for a three-degree-of-freedom parallel manipulator utilizing an optimally calibrated LQR-PID control architecture was conducted. Genetic algorithms (GA), particle swarm optimization (PSO), and grey wolf optimization

(GWO) were implemented to concurrently optimize both the LQR weighting coefficients and PID controller parameters.

The control systems were assessed through multiple performance metrics, including sum of squared errors (SSE), integral of squared error (ISE), integral of absolute error (IAE), and control-torque error quantification. While all three optimization methodologies demonstrated enhanced tracking performance relative to baseline approaches, GWO consistently produced the most favorable error metrics across all evaluation criteria.

Regarding the input link dynamics, the GWO-optimized controller achieved an SSE of 0.512, ISE of 0.0030, and IAE of 0.0213, substantially outperforming PSO results of 0.895, 0.0068, and 0.0672, respectively, and GA results of 1.395, 0.0110, and 0.1120. Furthermore, GWO demonstrated superior performance in both passive-link tracking and torque-control accuracy.
These findings establish GWO as the preeminent optimization technique among the three algorithms examined for the specified LQR-PID control framework. The resulting optimized controller delivers precise and smooth trajectory tracking with negligible residual error accumulation.

Subsequent research endeavors will examine controller robustness under payload variations, measurement noise contamination, external disturbance rejection, actuator saturation constraints, and validation through real-time experimental implementation.

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