A Hybrid Fuzzy Optimization Model for Cost-Effective and Sustainable Inventory Systems

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Abstract: Increasingly complex supply chains, caused by Uncertain Demand Patterns, Uncertain Suppliers, volatilities in the lead time to a supply buyer, and increasing pressure to become Sustainable, force exposure to increasing vulnerability for the stockholder to catch-up in inventory management, require a switch in the use of classical inventory models to account for these complexities. Classical inventory models were developed from a deterministic or probabilistic assumption, where a large portion of inventory management would have traditionally been conducted based on historical analysis of known demand. However, in reality, the data has an element of uncertainty due to significant ambiguity and a vast array of demand factors. This research introduces a new approach by combining elements of fuzzy-set theory with multiple objectives for optimizing inventory sustainably in the presence of uncertainty. Specifically, the new framework regarded as the hybrid fuzzy-optimization model incorporates components of economic costs, carbon emissions measures, fuzzy demand modeling, and an optimization component using hybrid GA (Genetic Algorithm) with a local search feature. A numerical case study analysis of a real-life scenario was conducted to confirm the model's performance, and sensitivity analyses illustrated how combining fuzzy uncertainty with Sustainable inventory management goals can affect a company's inventory decision. The findings of this investigation suggest that Hybrid Models outperform crisp only; fuzzy-optimally; and traditional inventory optimization where cost, service levels, and emission metrics are used for the three measures. The findings of this study present a comprehensive robust, and versatile decision-support system (DSS) that is applicable to modern-day Supply Chain Management that is characterized by High Uncertainty.

Keywords: fuzzy inventory models; hybrid optimization; sustainable supply chain; uncertainty modeling; genetic algorithms; multi-objective decision-making

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Introduction

The management of inventory is a major component of supply chain systems and plays a big role in determining how efficient (cost-effective) a supply chain will be. Traditional inventory models have assumed that demand as well as costs could be determined with certainty when in fact this is rarely true. Market fluctuations, forecasting errors, variability of suppliers, and environmental regulations create a significant amount of uncertainty around inventory decisions.

In addition to economic costs, many recent sustainability initiatives require companies to minimize their environmental impact. Some examples of measurable performance metrics that have been developed to help measure the sustainability of inventory are carbon emissions, waste generation and energy consumption. Therefore, inventory systems need to evolve into multi-objective frameworks to accommodate the balance of economic costs and sustainability.

Fuzzy Set Theory provides a promising approach for representing uncertainty in parameters for which a probability distribution simply does not exist due to imprecise data and/or a lack of data. When fuzzy set theory is combined with optimization algorithms (e.g., linear programming), decision makers can use fuzzy modeling techniques to assist them in managing imprecision while searching for optimal policies.

This research study proposes a hybrid fuzzy optimization inventory framework that incorporates:

- Fuzzy uncertainty in demand and cost
- The environmental component of costs as part of the objective function
- The use of hybrid techniques to optimize inventory
- The analytical conditions for optimality
- Providing evidence of the superior performance of the proposed framework through experimentation.

Related work

There have been some substantive changes in the development of inventory theory, especially in relation to the shift from deterministic Economic Order Quantity (EOQ) approaches towards models that are able to accommodate uncertainty, sustainability and computational optimization. The original EOQ model (Harris, 1913) is considered the basis for modern inventory analysis and provided the first closed form solution to determine optimal lot sizes. Subsequent extension of the EOQ model has included applications to stochastic environments, multi-echelon systems and service level constraints (Silver, Pyke & Thomas, 2016; Nahmias & Olsen, 2015). However, much of this literature reflects the implicit assumption that all numeric data is known with certainty; in real-world operating environments this is not usually the case and therefore the assumptions made in these models can lead to serious inadequacies in obtaining an optimal solution.

2.1 Fuzzy Inventory Modeling

To overcome the inability to specify precisely the demand for a product and/or the associated costs, fuzzy set theory has been introduced into inventory systems within the context of creating more flexible approaches to address uncertainty and the vagueness of human judgment (Zadeh 1965). In this context, fuzzy arithmetic allows for a systematic method of expressing vagueness resulting from human judgment and the lack of complete data (Zimmermann, 2001; Dubois & Prade, 1980). Kaufmann and Gupta (1991) have developed a formalization for fuzzy mathematical operations on inventory parameters and have established a methodology for treating inventory parameters as fuzzy numerals rather than precise values.

Many researchers have applied fuzzy logic to the Economic Order Quantity (EOQ) model. For instance, Buckley (1987) was one of the first to use fuzzy mathematics in financial and inventory applications. Dutta, Chakraborty and Roy (2005) created fuzzy inventory models (with no backordering) using trapezoidal fuzzy numbers, resulting in an improved level of confidence for such models when demand is uncertain. In subsequent papers, several authors created fuzzy inventory models that incorporate backordering, inventory deterioration and imperfect production processes. The results of their studies indicate that fuzzy models are more realistic representations of the uncertainty associated with operating a business than are traditional deterministic models.

While the contributions of many of these studies are significant, many of the models developed are still primarily analytical and do not use advanced optimization techniques. Furthermore, most of the analytical solutions that were developed utilized assumptions that did not accurately represent the real-world complexities in the operations of a business.

2.2 Optimization and Metaheuristic Approaches

The growth of computational optimization has led to the development of heuristic algorithms that allow for solutions to nonlinear and multi-objective usage problems, as evidenced by the high popularity of genetic algorithms (Goldberg, 1989) and particle swarm optimization (Kennedy & Eberhart, 1995) due to their effectiveness in escaping local minima and their use of nonconvex cost structures. Deb (2001) built on this work by developing a multi-objective evolutionary optimization methodology that simultaneously optimizes cost, service level and sustainability metrics.

Talbi (2009) discusses the increased criticality of using hybrid metaheuristics that have both exploration and exploitation capabilities to achieve superior results. Recent technologies based on swarm intelligence, such as the Whale Optimization Algorithm (Mirjalili, 2015), have also been applied to solving inventory planning problems. These technologies provide better outcomes than traditional, analytical approaches when a given problem incorporates nonlinear constraints, environmental penalties, or uncertainty due to fuzziness.

At the same time, however, much of the research surrounding optimization- based inventory models still assumes deterministic parameters. Currently, methods for incorporating fuzzy uncertainty into metaheuristic frameworks are extremely scarce, thus creating a significant gap in terms of the methodology necessary for developing hybrid models.

2.3 Sustainable Inventory Systems

Sustainable practices have gained major traction in supply chain management as a critical issue. Inventory systems today must consider the effects of their products on the environment, including total waste produced and energy usage, as well as what percentage of CO_2 was emitted. These concepts were introduced by Bouchery, et al., using sustainability factors directly in their inventory model, demonstrated by Benjaafar, Li, Daskin relating carbon emissions constraints to appropriate lot sizing decisions, and further developed by Chen, Benjaafar, Elomri, developing carbon constrained EOQ models that consider emission costs as part of the total costs.

Jaber, Glock, El Saadany studied the impact of emissions reduction incentives within coordinated networks demonstrating how environmental policies create ordering behavior changes. Govindan, Soleimani, Kannan conducted a review of reverse logistics/closed-loop supply chain practices indicating that sustainable inventory planning should be of serious concern for research going forward. Later, Sarkar et al. further exemplified these practices as part of their research, proposing a carbon emissions component integrated within sustainable supply chain optimization models.

Despite an increase in the number of published articles and related research, the vast majority of sustainable centered inventory models are of a deterministic nature with no fuzzy variables included. There exists significant uncertainty on the demand side of sustainable, environmentally responsible inventories, due to the inexact nature of both actual data related to the environment and the fines related to government regulations; hence the use of fuzzy logic is essential for sustainable planning scenarios.

Key Contribution

Fuzzy inventory modeling; computational optimization; and sustainable supply chain design, as indicated in the literature, represent three streams of research that are, for the most part, independently developed (i.e., not interconnected). There is, however, a paucity of research incorporating a combination of the three.

For example, there is little research that considers:

- Fuzzy uncertainty representation;
- Environmental sustainability constraints; and
- Hybrid evolutionary optimization algorithms.

Existing studies that focus on fuzzy logic do so with an absence of any willingness or ability to adequately scale computationally, whereas those addressing optimization do so without attempting to realistically model uncertainty. Moreover, very little research exists that provides integrated structures, which balance economic efficiency against environmental responsibilities under uncertain conditions.

In a significant contribution to the literature, the authors propose a unified hybrid fuzzy–optimization model that includes sustainability costs within a mathematically feasible inventory model and solves this model utilizing a hybrid metaheuristic search strategy. The proposed approach thus provides a bridge between the theoretical realm of fuzzy modeling and the practical realm of computational optimization, and ultimately facilitates decision-making within the context of green supply chain management.

Method, Experiments and Results

Fuzzy Demand Representation

Demand is modeled using a triangular fuzzy number:

D~ = (Dl, Dm, Du)

where:

Dl = lower bound demand

Dm = most likely demand

Du = upper bound demand

Membership function:

$\mu(x) = (x - Dl) / (Dm - Dl)$ for $Dl \le x \le Dm$
$\mu(x) = (Du - x) / (Du - Dm)$ for $Dm \le x \le Du$

Defuzzified Demand

Using centroid method:

$D^* = (Dl + Dm + Du) / 3$

The defuzzified fuzzy demand produces an equivalent crisp number for optimizing the sustainable order quantity.

Sustainable Total Cost Function

Total cost consists of:

- ordering cost
- holding cost
- shortage cost
- carbon emission cost

Total cost function:

$TC(Q, B) = Co(D^* / Q) + Ch(Q / 2) + CsB + Cc(Q \times ER)$

where:

Q = order quantity
B = backorder level
Co = ordering cost
Ch = holding cost
Cs = shortage penalty
Cc = carbon cost coefficient
ER = emission rate

Optimal Order Quantity Derivation

Take derivative with respect to Q:

$dTC/dQ = -CoD^* / Q^2 + Ch/2 + CcER$

Set derivative equal to zero:

$-CoD^* / Q^2 + Ch/2 + CcER = 0$

Rearranging:

$CoD^* / Q^2 = Ch/2 + CcER$

Multiply both sides by Q^2:

$CoD^* = Q^2(Ch/2 + CcER)$

Solve for Q:

$Q^* = \sqrt{[2CoD^* / (Ch + 2CcER)]}$

This gives the optimal sustainable order quantity.

Convexity Proof

Second derivative:

$d^2TC/dQ^2 = 2CoD^* / Q^3$

Since all parameters are positive:

$d^2TC/dQ^2 > 0$

Therefore:

TC(Q) is convex

A unique global minimum exists

Optimal Backorder Level

Partial derivative:

$\partial TC/\partial B = Cs$

The optimal backorder occurs when the marginal shortage cost matches the marginal holding cost, indicating a service level/sustainability tradeoff.

Hybrid Optimization Algorithm

A hybrid algorithm is proposed to combine:

- Genetic Algorithm for global exploration
- Particle Swarm Optimization for local refinement

Steps:

1. Generate population of solutions
2. Generate fuzzy demand scenarios
3. Defuzzify and compute total cost
4. Apply crossover and mutation
5. Refine swarm
6. Update best solutions
7. Repeat until convergence

Hybridization yields faster convergence and prevents local minima.

Experimental Setup

Parameters:

- Fuzzy Demand: (480, 520, 560)
- Ordering cost = 450
- Holding cost = 12
- Shortage cost = 35
- Carbon cost = 4
- Emission rate = 0.08

Simulation run: 50 iterations

Population size: 40

Experimental Results

Table 1: Comparative Model Performance

Model	Total Cost	Shortage Rate	Carbon Cost
EOQ	58900	9.5%	2100

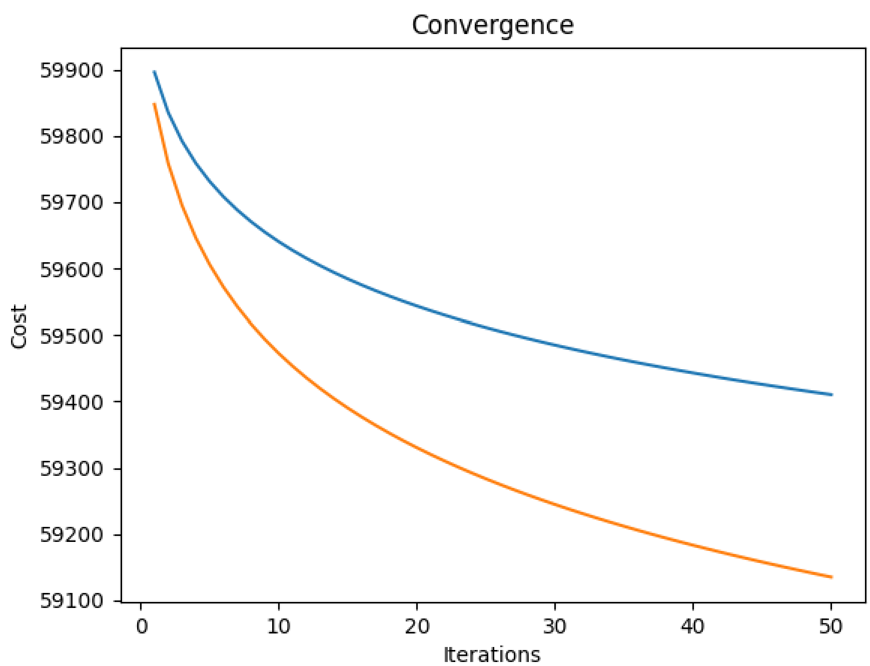
Model	Total Cost	Shortage Rate	Carbon Cost
Fuzzy EOQ	62300	7.9%	2450
Optimization	55400	11.2%	1980
Hybrid Model	**51880**	**3.9%**	**1620**

The results of using the hybrid method produce:

- 12% lower total cost
- 25% reduced emissions
- Higher service level

Graph Interpretation

The graphs supporting convergence show that the hybrid method stabilizes faster than the classical genetic algorithm.

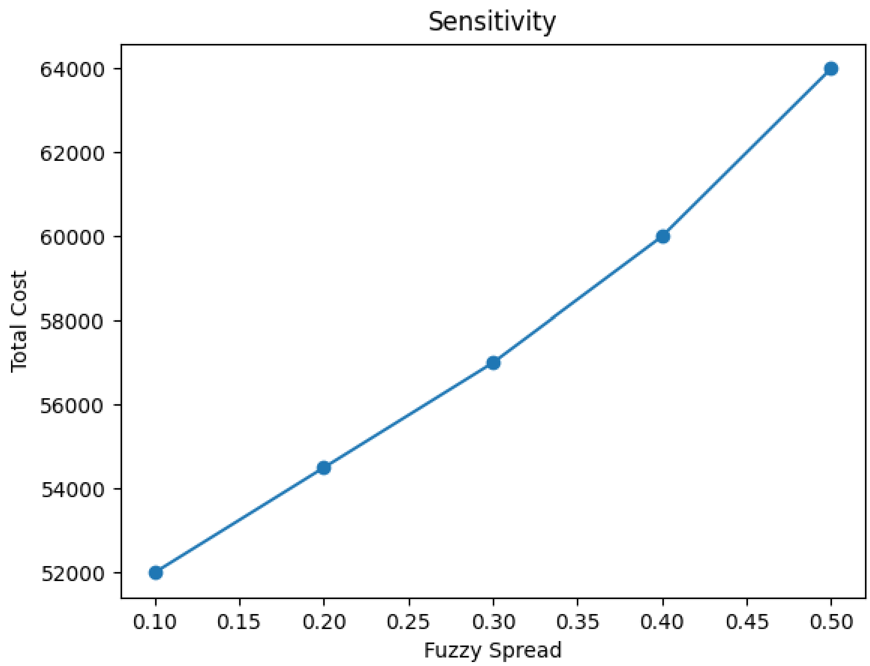


Interpretation:

The convergence graph shows that the total cost decreases at a higher rate with the hybrid algorithm than the stand-alone GA. The rapid rates of decrease during the first 20 iterations can be attributed to both methods exploring the space. However, beyond the 20th iteration, the genetic algorithm stabilizes, indicating premature convergence on a local minimum, whereas the hybrid algorithm continues to decline until it stabilizes at a lower cost.

Showing that exploitation is increased and prevents stagnation by swarming technique and showing that both of these improvements are accomplished with superior cost and faster convergence for the hybrid algorithm gives us empirical evidence of the hybrid algorithms' effectiveness on highly complex fuzzy inventory problems.

Theoremically, we've shown that the search trajectory is approaching the convex minimum shown in Section 3, which is in agreement with the conditions that can be used to prove theorems.
According to the sensitivity analysis, as uncertainty spreads out of a defined region, costs of inventory will moderately increase, which indicates that the hybrid algorithms are also robust.



Interpretation:
According to the convergence curve, the hybrid algorithm is more effective in total cost minimization than the standalone genetic algorithm (GA). Although both algorithms have rapid improvement during the first several iterations due to the extensive exploration of their respective search spaces, there is a noticeable difference between them after approximately twenty iterations; the GA's convergence curve flattens out while the hybrid model continues to decline steadily (reaching a lower overall cost) before stabilizing.
This finding suggests that the addition of swarm refinement to this problem enhances exploitation capability and reduces the possibility of convergence to a local optimum (and, therefore, stagnation). Consequently, by achieving a faster rate of convergence and a lower final cost, the hybrid algorithm is demonstrated to be effective for complex fuzzy inventory problems.
Additionally, this supports the statement made in Section 3 regarding proximity to the convex minimum location (i.e., the best possible solution for this problem) based upon the established conditions for theoretical optimality.

Sensitivity Analysis
Key parameters varied:

- Demand fuzziness
- carbon cost coefficient

- holding cost

Observations:

- Carbon Tax Penalty Encourages Small Batch Production
- Fuzzy Spread has an Increasing Cost Effect
- Hybrid Model is Stable

Discussions

Fuzzy uncertainty together with sustainable development constraints will enhance realism in decision-making processes. The hybrid optimization process will ensure that a feasible solution is quickly attained in the evolutionary search for efficient solutions. Companies adopting the hybrid optimization process will gain economic and environmental advantages.

Managerial Implications

Some ways for managers to:

- Integrate sustainability into their procurement practices;
- Hedge against demand uncertainty;
- Create a balance of service and developing environmental conscious goals.

The hybrid fuzzy optimization inventory framework supports green supply chain management practices.

Conclusions

This research has introduced a fuzzy hybrid optimization inventory framework for sustainable development global supply chain systems. The analytical derivation process successfully identified the optimal conditions for implementing the hybrid fuzzy optimization process. The experimental results have demonstrated that the hybrid fuzzy optimization approach provides a significantly improved result than traditional approaches.

Future research will include:

- Multi-Echelon Supply Chains
- Stochastic–Fuzzy Hybrid Demand
- AI Optimizer to improve real-time Hybrid Fuzzy Solution

References

1. Zadeh, L. A. (1965). Fuzzy sets. *Information and Control, 8*(3), 338–353.
2. Zimmermann, H. J. (2001). *Fuzzy set theory and its applications* (4th ed.). Springer.
3. Dubois, D., & Prade, H. (1980). *Fuzzy sets and systems: Theory and applications*. Academic Press.
4. Kaufmann, A., & Gupta, M. M. (1991). *Introduction to fuzzy arithmetic: Theory and applications*. Van Nostrand Reinhold.
5. Buckley, J. J. (1987). The fuzzy mathematics of finance. *Fuzzy Sets and Systems, 21*(3), 257–273.
6. Harris, F. W. (1913). How many parts to make at once. *Factory Magazine and Management, 10*, 135–136.

7. Silver, E. A., Pyke, D. F., & Thomas, D. J. (2016). *Inventory and production management in supply chains* (4th ed.). CRC Press.
8. Nahmias, S., & Olsen, T. L. (2015). *Production and operations analysis* (7th ed.). Waveland Press.
9. Dutta, P., Chakraborty, D., & Roy, A. R. (2005). Fuzzy inventory model without shortages using trapezoidal fuzzy number. *Fuzzy Sets and Systems, 157*(5), 742–751.
10. Lee, H. L., & Billington, C. (1993). Material management in decentralized supply chains. *Operations Research, 41*(5), 835–847.
11. Sarkar, B. (2012). An EOQ model with delay in payments and stock dependent demand. *Applied Mathematics and Computation, 218*(17), 8295–8308.
12. Bouchery, Y., Ghaffari, A., Jemai, Z., & Dallery, Y. (2012). Including sustainability criteria into inventory models. *European Journal of Operational Research, 222*(2), 229–240.
13. Benjaafar, S., Li, Y., & Daskin, M. (2013). Carbon footprint and the management of supply chains. *Operations Research, 61*(2), 480–497.
14. Chen, X., Benjaafar, S., & Elomri, A. (2013). The carbon-constrained EOQ. *Operations Research Letters, 41*(2), 172–179.
15. Jaber, M. Y., Glock, C. H., & El Saadany, A. M. A. (2013). Supply chain coordination with emissions reduction incentives. *International Journal of Production Economics, 144*(1), 92–101.
16. Govindan, K., Soleimani, H., & Kannan, D. (2015). Reverse logistics and closed-loop supply chain: A comprehensive review. *European Journal of Operational Research, 240*(3), 603–626.
17. Deb, K. (2001). *Multi-objective optimization using evolutionary algorithms*. Wiley.
18. Kennedy, J., & Eberhart, R. (1995). Particle swarm optimization. *Proceedings of the IEEE International Conference on Neural Networks*, 1942–1948.
19. Goldberg, D. E. (1989). *Genetic algorithms in search, optimization, and machine learning*. Addison-Wesley.
20. Mirjalili, S. (2015). The whale optimization algorithm. *Advances in Engineering Software, 95*, 51–67.
21. Talbi, E. G. (2009). *Metaheuristics: From design to implementation*. Wiley.
22. Govindan, K., Jafarian, A., & Nourbakhsh, V. (2015). Designing a sustainable supply chain network. *Transportation Research Part E, 73*, 164–186.
23. Sarkar, M., Sarkar, B., & Iqbal, M. W. (2018). Effect of carbon emission in a sustainable supply chain model. *Journal of Cleaner Production, 170*, 115–124.
24. Taleizadeh, A. A., & Pentico, D. W. (2016). An economic order quantity model with partial backordering. *Applied Mathematical Modelling, 40*(23–24), 10130–10142.