Control-Oriented State-Space Reconstruction of Clinical EEG Signals for Dynamical Brain Modeling

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Abstract: Electroencephalography (EEG) signals contain a population of neurons in their aggregate electrical activity, and are commonly classified and diagnosed in statistical and deep learning approaches. Nonetheless, the majority of current methods treat EEG as a data-oriented pattern recognition problem rather than as the measurable behavior of a nonlinear dynamical system. This view restricts the construction of control-based schemes for analyzing neural modulation and stability. The paper provides a control-based state-space rebuilding model of clinical EEG signals based on nonlinear dynamical systems theory. The development of a mathematical representation of delay-coordinate embedding and nonlinear state-evolution modeling of EEG recordings as scalar observations of the unobservable neural state variables allows us to formulate a mathematical representation of the neural state. The formulation offered enables reconstruction of phase space, definition of attractors, and interpretation of brain dynamics in terms of stability. In contrast to traditional black-box methods, the framework provides a theoretical basis for observer design and closed-loop neural control strategies. The paper describes a systematic approach to integrating dynamical EEG modeling with future control and intervention strategies.

Keywords: Electroencephalography (EEG); State-space reconstruction; Nonlinear dynamical systems; Control-oriented modeling; Brain dynamics; Phase-space analysis; Neural signal processing; Dynamical stability.

Introduction

Electroencephalography (EEG) is also widely used for monitoring cortical activity in both clinical diagnostics and brain-computer interfaces. Recent developments in deep learning have significantly enhanced the performance of automated EEG classification and decoding systems [1], [3] to [6]. The overall literature reviews show that modern EEG studies are mainly aimed at improving classification accuracy using increasingly sophisticated neural networks [1], [2]. Similar advances in the signal processing techniques have equally enhanced EEG analysis pipelines [7], as the scale of real-time neural monitoring has been extended by advances in acquisition and feedback technologies [8]. Moreover, functional and effective connectivity studies examine interregional neural interactions using EEG recordings [9]. Although these developments have been achieved, most frameworks treat EEG as a high-dimensional pattern recognition problem rather than as the measurable output of a nonlinear dynamical system. Neural oscillations result from nonlinear interactions between thalamocortical and cortical networks [10]. Nevertheless, a systematic state-space reconstruction and control-based modeling of EEG signals is not well studied. A dynamical systems view allows us to characterize attractor geometry, stability properties, and nonlinear transitions that cannot be easily obtained by purely statistical methods. To fill this gap, this paper proposes a minimalist control-based state-space reconstruction framework for clinical EEG signals based on nonlinear dynamical systems theory. The reformulation of EEG analysis in the context of state-space modeling provides a basis for stability analysis and subsequent closed-loop strategies for neural modulation. The article offers a control-based view of EEG signal analysis by combining concepts from nonlinear dynamical systems and state-space reconstruction. It is proposed that the framework will close the gap between conventional data-driven methods and interpretable dynamical modelling, and will enable analysis of stability and the design of future strategies for closed-loop neural modulation.

Nonlinear Dynamical Formulation and State-Space Reconstruction

Electroencephalographic (EEG) signals may be considered visual representations of an incomprehensible neural dynamical system. Suppose the obscure neural conditioning is represented as,

$$\mathbf{z}(t) \in \mathrm{R}^n \tag{1}$$

Nonlinear evolution that is controlled by

$$\dot{\mathrm{z}}(t) = \mathrm{F}\big(\mathrm{z}(t)\big) \tag{2}$$

where $\mathrm{F}(\cdot)$ refers to an unknown nonlinear vector field that functions to describe the interaction between neuronal populations. The EEG signal $x(t)$, which has been measured, can be described as.

$$x(t) = h(\mathrm{z}(t)) + \eta(t) \tag{3}$$

where $h(\cdot)$ is a measurement function and $\eta(t)$ is a measurement noise. Since it is impossible to get access to the internal state $\mathrm{z}(t)$, a reconstruction must be performed using the known scalar time series progression $x(t)$ only. Delay-coordinate embedding: A mathematically-grounded approach to retrieving phase-space structure of the underlying system. An individual who has been rebuilt at the time t It is said to be in a state.

$$\mathrm{X}(t) = [x(t), x(t-\tau), x(t-2\tau), \ldots, x(t-(m-1)\tau)] \tag{4}$$

where τdenotes the embedding delay and $m$the embedding dimension.
When the embedding conditions are satisfied, the reconstructed state space retains the topological properties of the original dynamical system. As a result, the investigation of such qualitative characteristics as attractor geometry, invariant manifolds, and stability behavior can be performed directly based on the reconstructed trajectory. With this formulation, EEG analysis is no longer a signal-processing paradigm, but a problem of dynamical systems modeling. Rather than finding class labels as a function of inputs, the task is to discover the nonlinear evolution that plays out in reconstructed space:

$$\dot{\mathrm{X}}(t) = \mathrm{G}\big(\mathrm{X}(t)\big) \tag{5}$$

and $\mathrm{G}(\cdot)$ is an approximation to the unknown system dynamics in observed coordinates. This representation enables control-oriented analysis later, such as stability analysis and the design of interventions based on the model.

Control-Oriented Representation and Stability Interpretation

When the EEG signal is reconstructed in state space, the dynamics can be written in control-affine form, allowing analysis of modulation. The state vector that has been rebuilt is $\mathrm{X}(t) \in \mathbb{R}^m$. We introduce the extension of autonomous formulation by external or endogenous modulation input $u(t)$:

$$\dot{\mathrm{X}}(t) = \mathrm{G}(\mathrm{X}(t)) + \mathrm{B}u(t) \tag{6}$$

where:

- $\mathrm{G}(\mathrm{X})$represents intrinsic nonlinear brain dynamics,
- B is an input distribution matrix,
- $u(t)$models external neuromodulatory intervention (e.g., stimulation) or internal regulatory influences.

This expression enables EEG activity to be viewed as a nonlinear dynamical system, rather than a passive process of observation. Table 1 summarizes the current methods of EEG analysis and their limitations comparatively.

Table 1. Comparison of conventional EEG analysis methods and the proposed framework (compiled from [1]–[6], [9]).

Approach	Key Focus	Limitation	Proposed Contribution
Deep Learning Methods	Classification accuracy	Black-box modeling, lack of interpretability	Interpretable dynamical modeling
Signal Processing Techniques	Feature extraction	No explicit system representation	State-space formulation
Connectivity Analysis	Brain region interactions	Limited control-oriented insights	Control-affine modeling
Proposed Framework	Dynamical system representation	—	Stability analysis and control-oriented EEG modeling

Local Stability Interpretation

Let X^*denote an equilibrium point such that

$$\mathrm{G}(\mathrm{X}^*) = 0 \tag{7}$$

Linearizing around X^*, we obtain

$$\delta\dot{\mathrm{X}}(t) = \mathrm{J}(\mathrm{X}^*)\delta\mathrm{X}(t) \tag{8}$$

where $\mathrm{J}(\mathrm{X}^*)$is the Jacobian matrix evaluated at the equilibrium.
The eigenvalues of J determine local stability properties:
Real parts with negative real parts are attracted to a stable state. Real parts that are positive lead to unstable dynamics. Complex eigenvalues - oscillatory behaviour. Modeling in clinical EEG. In clinical EEG, the transition across regimes of stability can be linked to pathological phenomena, including the occurrence of a seizure or a nonnormal rhythmic entraining process.

Implications for Closed-Loop Neural Modulation

The reconstruction of EEG into a control-based state-space formulation enables stability-based seizure prediction, adaptive neurostimulation design, feedback-based modulation strategies, and observer-based state estimation. This method of analysis is not based on post hoc or classification of signals, but rather enables predictive, intervention-directed analysis of neural dynamics. It follows that EEG may be construed as a quantifiable response of some latent nonlinear dynamical system that can be systematically controlled, stabilized, and directed by principled control-theoretic techniques.

Discussion: Toward Dynamical Brain System Engineering

Recent research on EEG analysis has mostly focused on machine-learning-based classification systems [2], [3], [6], [8]. Although these methods exhibit excellent predictive power, they tend to assume that EEG signals are high-dimensional time series rather than expressions of structured nonlinear dynamical processes. In-depth surveys have focused on developments in deep learning and feature engineering [4], [5], but little attention has been paid to explicit control-based mathematical modeling. Thalamocortical dynamics have been previously analyzed nonlinearly using models, demonstrating the applicability of dynamical systems theory in explaining pathological oscillations [1]. Equally, studies have examined functional and effective connectivity [10], showing that brain activity inherently has a networked dynamical structure. Nonlinear dynamics to a control-theoretic interpretation Unified state-space reconstruction framework Unification of nonlinear dynamics has not been much explored, however. This gap is filled by the formulation suggested in this work, which: 1. Takes the EEG as an observable of a latent nonlinear state system. 2. Uses delay-coordinate embedding of phase-space reconstruction. 3. Representing reconstructed dynamics as control-affine. 4. The analysis of pathological transitions in terms of stability. This view changes EEG studies to dynamical systems engineering of the brain. Theoretical foundations for closed-loop neurostimulation, Adaptive seizure control, Stability-based biomarkers, and Observer-Based neural state estimation can be found in such a framework. In addition, hybrid structures combining explainable recurrent modeling strategies [7] with state-space reconstruction can potentially allow combining interpretability with data-driven adaptability. A proposed control-oriented dynamical perspective therefore supplements, rather than substitutes for, modern deep learning paradigms with a mechanistic basis for clinical EEG interpretation. The general workflow of the proposed framework is shown in Figure 1.

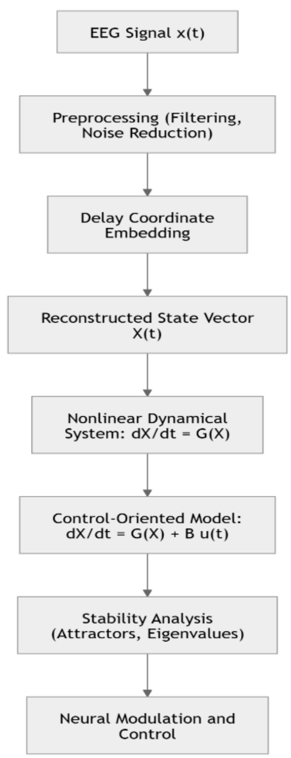

Figure 1. Proposed control-oriented state-space reconstruction framework for EEG signal modeling based on nonlinear dynamical systems theory.

Conclusions

The paper introduced a nonlinear dynamical control-based approach to effective clinical EEG signal analysis via state-space reconstruction. The study redefines EEG analysis by treating EEG recordings as visible projections of latent neural dynamics, rather than as data-driven phenomena, within a framework of structured mathematical systems. Pathological transitions in brain activity are understood in a principled manner by combining delay embedding, nonlinear state-evolution modeling, and stability-based interpretation. The suggested framework provides a conceptual basis for the future progress of observer-based neural state estimation and closed-loop neuromodulation. Relating nonlinear dynamics and control theory to computation-based modern methods is an exciting prospect for dynamical brain system engineering in the field of clinical neurophysiology.

References

1. Cao, J., Zhao, Y., Shan, X., et al. (2021). Brain functional and effective connectivity based on electroencephalography recordings: A review. Human Brain Mapping, 43(2), 860–879. https://doi.org/10.1002/hbm.25683
2. Chaddad, A., Wu, Y., Kateb, R., & Bouridane, A. (2023). Electroencephalography signal processing: A comprehensive review and analysis of methods and techniques. Sensors, 23(14), 6434. https://doi.org/10.3390/s23146434
3. He, F., Sarrigiannis, P. G., Billings, S. A., et al. (2016). Nonlinear interactions in the thalamocortical loop in essential tremor: A model-based frequency domain analysis. Neuroscience, 324, 377–389. https://doi.org/10.1016/j.neuroscience.2016.03.028
4. Li, Y., Chen, E., Xiao, X., Xu, M., & Ming, D. (2025). Lightweight deep learning models for EEG decoding: A review. Journal of Neural Engineering, 22(6), 061004. https://doi.org/10.1088/1741-2552/ae2717
5. Lotte, F., Bougrain, L., Cichocki, A., et al. (2018). A review of classification algorithms for EEG-based brain–computer interfaces: A 10-year update. Journal of Neural Engineering, 15(3), 031005. https://doi.org/10.1088/1741-2552/aab2f2
6. Qin, Y., Zhang, Y., Zhang, Y., Liu, S., & Guo, X. (2023). Application and development of EEG acquisition and feedback technology: A review. Biosensors, 13(10), 930. https://doi.org/10.3390/bios13100930
7. Roy, Y., Banville, H., Albuquerque, I., Gramfort, A., Falk, T. H., & Faubert, J. (2019). Deep learning-based electroencephalography analysis: A systematic review. Journal of Neural Engineering, 16(5), 051001. https://doi.org/10.1088/1741-2552/ab260c
8. Usgaonkar, S. S., Edla, D. R., & Dharavath, R. (2026). Optimizing deep CNN architecture via hybrid Harris hawks arithmetic algorithm for EEG meditation classification. Neuroscience, 598, 100–110. https://doi.org/10.1016/j.neuroscience.2026.02.001
9. Wang, X., Liesaputra, V., Liu, Z., Wang, Y., & Huang, Z. (2023). An in-depth survey on deep learning-based motor imagery electroencephalogram (EEG) classification. Artificial Intelligence in Medicine, 147, 102738. https://doi.org/10.1016/j.artmed.2023.102738
10. Zhang, S., Wu, L., Yu, S., et al. (2024). An explainable and generalizable recurrent neural network approach for differentiating human brain states on EEG dataset. IEEE Transactions on Neural Networks and Learning Systems, 35(6), 7339–7350. https://doi.org/10.1109/TNNLS.2022.3214225