

Recent Studies based on Differential Equation Models and Nonstandard Finite Difference Methods for Infectious Disease Dynamics

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Abstract: Mathematical modeling developed a significant tool to understand spread and control infectious diseases. Numerous epidemic eruptions for example COVID-19, malaria, tuberculosis, hepatitis and Zika virus have exposed need for precise and dependable mathematical technique. Though, multiple usual numerical approaches fail to preserve positivity, stability and biological steadiness throughout simulations. Such studies summarize current growths within the field of differential equation models with nonstandard finite difference (NSFD) techniques which is mostly helpful in infectious disease dynamics.

Our study examines ordinary differential equation models, reaction-diffusion systems with dynamically reliable NSFD systems functional to dissimilar epidemic complications. Current findings helpful for NSFD techniques gives the better stability, preserve epidemiological properties and produce reliable numerical solutions associated with standard discretization methods. Our study also focusses role of vaccination, delay effects, diffusion, optimal control strategies in the epidemic modeling. Such methods have important applications like disease prediction, healthcare planning, epidemic prevention with public health decision-making.

Keywords: Numerical simulation; Reaction-diffusion models; Infectious diseases; Mathematical modeling; Differential equations; Nonstandard finite difference method; Epidemic dynamics.

Introduction

Infectious diseases have become one of major issue in universal public health. Diseases like influenza, malaria, tuberculosis, hepatitis, dengue, Ebola, HIV/AIDS, Zika virus and COVID-19 affect millions of nations globally. Fast infection transmission, mutation pathogens and population mobility disease control become tough. As this, mathematical modelling become more significant tool for studying disease span and forecasting epidemic performance [1, 2].

Mathematical epidemiology uses differential equations to define the disease transmission within the population. Ordinary differential equations (ODEs) with partial differential equations (PDEs) are mostly taken because it explains temporal and spatial changes within the disease dynamics field. Trejos et al. [30] given mathematical models which work on biological observations into the quantitative data sets, can be useful within the public healthcare preparation. Such technique

commonly used to estimate infection rates, which estimate involvement strategies and forecast future component and its outbreaks in **Table.1**.

The conventional susceptible infected recovered (SIR) widespread model presented infectious disease modelling. As, different generalization of the compartmental models for example SEIR, SIRS and reaction diffusion epidemic systems advanced for different diseases **Table.1**. Latest, studies attentive not only on disease transmission but also vaccination, media awareness, quarantine, treatment effects with memory-dependent transmission methods [7, 9, 16].

Table 1 Major Components from the Infectious Disease Modelling systems

Component	Description	Applications
Mathematical modelling	Construction from ODE and PDE epidemic structures	Disease forecast
Qualitative analysis	Stability, positivity, boundedness	Long-term epidemic behaviour
Numerical methods	Standard and NSFD systems	Numerical simulation
Practical applications	Public health planning and interventions	Vaccination and outbreak control

Other than this COVID-19, Multiple authors advanced epidemic technique for Ebola, malaria, tuberculosis, HIV/AIDS, hepatitis, dengue, and Zika virus kind of infections. Iqbal et al. [7] studied a SEIR epidemic model with memory effects to the Ebola transmission. Jamal et al. [8] gives a Zika virus technique which helps vertical transmission and Wolbachia-infected mosquitoes.

Musyoki et al. [15] analyzed the tuberculosis and diabetes co-infection dynamics with the help of a nonstandard finite difference (NSFD) scheme. Hoang [3,4] advanced numerical approximations to hepatitis B epidemic models and then analyzed their dynamical behavior.

Nonstandard finite difference schemes have become very useful in the infectious disease modelling. Standard numerical approaches sometimes fail to preserve positivity, boundedness with stability properties for epidemic structures. Mickens [28] presented theory for NSFD structures for such limitations. Afterword, Patidar [29] examined multiple latest expansions in NSFD techniques and their applications within the biological systems.

Literature Review and Background

Many recent studies established NSFD systems gives dynamically reliable solutions to the epidemic technique. Hoang and Ehrhardt [5] advanced dynamically reliable NSFD system to the extended SEIR epidemic model. Khan et al. [10] used NSFD technique for coronavirus transmission dynamics. Khalsaraei [11] given the positive and elementary stable NSFD system for influenza disease modelling. Treibert et al. [23] advanced NSFD system to forecasting COVID-19 dynamics with the help of SVICDR model **Table.2**.

Table 2 Common Infectious Diseases and Modelling Approaches

Disease	Common Model Type	Selected References
Ebola	SEIR epidemic models	[7, 21]
Malaria	Host–vector models	[1, 18]
Hepatitis B	Nonlinear ODE systems	[3, 4]
Tuberculosis	Co-infection models	[15]
Zika virus	Vector transmission models	[8, 14, 19]
COVID-19	SEIR and fractional models	[9, 10, 13, 23]

Mathematical modelling used widely and applied to different infectious diseases with the help of deterministic and numerical methods **Table.2**. Researchers mainly focused on compartmental ODE and PDE structures for disease transmission, stability behavior with intervention policies [30].

COVID-19

COVID-19 modelling enhanced as a most active research studies after pandemic outbreak. Kavya et al. [9] examined role of vaccination with booster doses within the controlling Omicron variant. Khan et al. [10] learned coronavirus transmission with the help of nonstandard finite difference (NSFD) system. Treibert et al. [23] suggested an NSFD system for SVICDR COVID-19 model. Liu et al. [13] later introduced fractional-order modelling for memory effects of the SARS-CoV-2 dynamics.

Ebola Disease

Ebola transmission also studied with help of SEIR and information-dependent epidemic structures. Iqbal et al. [7] advanced an SEIR epidemic model using memory effects for Ebola-like infections. Tadmon and Kengne [21] examined Ebola transmission dynamics with voluntary isolation and awareness policies with the help of NSFD approaches.

Hepatitis B Infection

Hepatitis epidemic models become most of important as its complex transmission behavior. Hoang [3] offered consistent numerical estimates for hepatitis B virus model. Later, Hoang [4] examined extended hepatitis B epidemic scheme with dynamically consistent discrete model.

Through COVID-19 pandemic, mathematical models established significant amount of attention because of its estimate transmission rates, hospitalization levels with vaccination efficiency. Kavya et al. [9] reviewed influence of vaccination and booster doses for Omicron variant with the help of mathematical modelling methods in **Figure 1** and **Figure 2**. Naz et al. [16] studied vaccination technique with their role for analyzing COVID-19 intervention policies. Like as, Liu et al. [13] examined dynamics for SARS-CoV-2 Omicron variant with the help of fractional-order modelling methods memory effects.

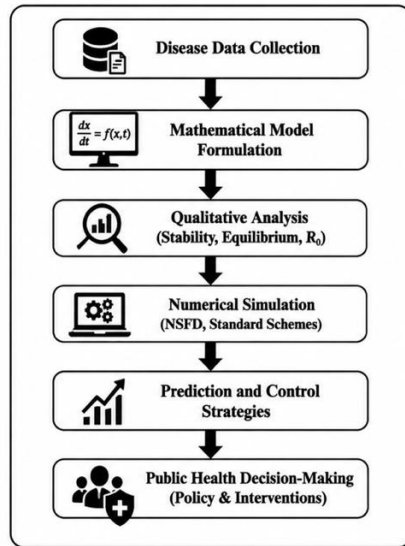


Figure 1. General framework of infectious disease modelling. [30]

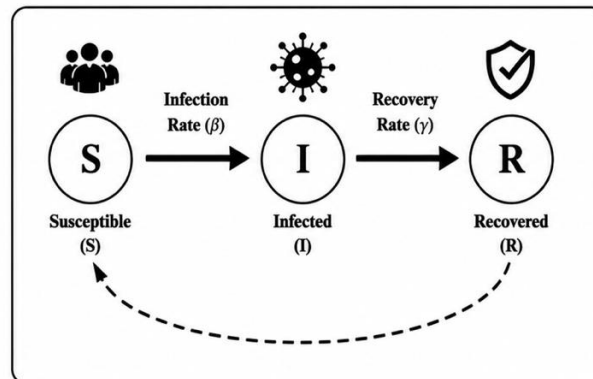


Figure 2. Basic structure of the SIR epidemic model. [30]

Numerical Methods and Models

Malaria Modelling

Malaria become the one of important vector-borne diseases studied using mathematical epidemiology. Faragó and Mosleh [1] examined qualitative properties for discrete like malaria propagation models. Sarkar et al. [18] learnt malaria dynamics using the media awareness and saturated treatment effects with the help of NSFD schemes.

Tuberculosis Models

Tuberculosis models mostly contain latent infection phases with co-infection effects. Musyoki et al. [15] advanced a tuberculosis diabetes co-infection model and analyzed it with the help of nonstandard numerical approaches.

Zika Virus Models

Numerous studies dedicated to the vector-host communication and vertical spread of Zika virus. Jamal et al. [8] planned mathematical model for Zika virus transmission including Wolbachia-infected mosquitoes. Maamar et al. [14] advanced time-fractional Zika virus transmission model with the help of NSFD system. Same time, Sarkar et al. [19] also examined Zika dynamics using information dependent transmission and control strategies.

Influenza Models

Influenza epidemic methods are extensively helpful for the stability and numerical analysis trainings. Khalsaraei [11] suggested positive and elementary steady NSFD system for influenza disease modelling.

HIV/AIDS Models

HIV/AIDS epidemic dynamics also been explored with the help of nonlinear differential equation systems. Li et al. [12] considered impact of the standard and nonstandard finite difference systems based on the HIV dynamical models with compared numerical behaviors.

Reaction–Diffusion and Delayed Epidemic Models

Reaction diffusion structures define the spatial movement and disease spread. Pasha et al. [17] examined NSFD methods to reaction diffusion epidemic structures. Xu et al. [26] planned nonstandard numerical method behind and diffusive viral infection techniques. Yang et al. [27] explored within host virus dynamics for virus to cell with cell-to-cell transmission.

General NSFD and Numerical Modelling Studies

Mickens [28] presented theoretical establishment for nonstandard finite difference arrangements for biological and epidemic schemes. Patidar [29] studied latest changes within the NSFD methods and its applications. Songolo and Bidégaray-Fesquet [20] later generalized multiple NSFD rules for nonlinear ODE schemes. Takács et al. [22] advanced high-order numerical approaches to the epidemic models using different recruitment rates.

Standard and Nonstandard Numerical Methods

Multiple epidemic differential equation models we cannot deal analytically. So that, numerical methods are significant to get estimated solutions then we easily get understanding disease dynamics [28,29]. Major numerical methods for example Euler, Taylor, Runge–Kutta and finite difference methods mostly helpful for solving ordinary differential equations (ODEs) with partial differential equations (PDEs) within the field of infectious disease modelling [1,5].

For epidemic model,

$$\frac{dx}{dt} = F(x, t),$$

And the forward Euler scheme we know,

$$x_{n+1} = x_n + hF(x_n, t_n),$$

whereas ($h > 0$) be the time step size.

To avoid such problems, Mickens gives nonstandard finite difference (NSFD) technique [28]. NSFD systems, we write as

$$\frac{x_{n+1} - x_n}{\phi(h)} = F(x_n, x_{n+1}),$$

whereas,

$$\phi(h) = h + O(h^2).$$

The NSFD method gives vital dynamical conduct for the epidemic techniques completely independent from step size [2,3] **Table 3**. Most of latest studies completely applied in to the NSFD schemes, COVID-19, malaria, HIV/AIDS, Ebola, hepatitis, tuberculosis and Zika virus representations [4,7,10,14,15,21,23].

Table 3 commonly used numerical methods within the infectious disease modelling.

Numerical Method	Main Property	Applications
Euler Method	Simple implementation	Basic epidemic simulation
Runge–Kutta Method	Higher accuracy	ODE epidemic models
Finite Difference Method	PDE approximation	Reaction–diffusion systems
NSFD Method	Positivity and stability preservation	Long-term epidemic dynamics

The development is consistent numerical methods **Table 3** becomes most active in the research of mathematical epidemiology [5,20,22].

Discussions

Mathematical models mostly established from ODEs and PDEs which are extensively used for study infectious disease transmission with outbreak forecast [5,10,15]. Latest research helps that NSFD approaches gives mainly stable and biologically meaningful mathematical solutions to the epidemic systems for example COVID-19, malaria, Ebola with hepatitis models [2,3,14,21]. Such approaches preserve positivity with stability compared to standard numerical systems [28,29].

Multiple scholars also applied epidemic models for the reaction diffusion schemes, vector-borne diseases, and fractional-order techniques [13,17]. Though, model accuracy completely depends on the epidemiological data sets with parameter approximation.

Conclusions

This study examined current growths within the deterministic infectious disease models with NSFD numerical methods. Differential equation models become mostly effective tools to analyze epidemic dynamics, disease control strategies and outbreak prediction [1,5,30].

Our work confirmed that NSFD methods mainly has domain significant dynamical properties for example positivity, boundedness with stability [2,28]. Further research should emphasis on the stochastic models, artificial intelligence incorporation with real-time epidemic forecast approaches to the public health applications.

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