Evolutionary Algorithms Based Optimal Tuning of LQR–PID Controller-A Review Study

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Abstract: The control of trajectory tracking for multi DOF parallel manipulators is extremely difficult due to the presence of high coupling factors and nonlinear dynamics. For these systems, Hybrid LQR-PID controllers are very promising in terms of robustness and tracking, and are therefore widely adopted. This paper analyzes the recent trend in the LQR-PID controller tuning using evolutionary algorithms, namely GA, PSO, DE, WOA, FA, BA, SCA, ABC, BBO, and GWO. It emphasizes the performance issues and identifies the missing areas in existing research.
Keywords: Parallel Manipulator; Trajectory Tracking; LQR–PID; Evolutionary Algorithms; GWO

Introduction

Researchers have increasingly focused on parallel manipulators because compared to serial manipulators, they are noticeably better in stiffness, accuracy, and dynamic performance [1]–[3]. 3-DOF parallel manipulators are utilized in several different applications including flight simulators, precision positioning systems, and medical robotics because they can provide the necessary accuracy and repeatability [4], [5]. Due to the structures of these manipulators, which consist of several closed-loop kinematic chains, manipulate systems improve stiffness and their ability to bear loads. This architecture, however, poses a significant problem to both modeling and control because of the strong coupling of the links and the closed-loop systems resulting in nonlinear dynamics [6]–[8].
Tracking a desired trajectory is one of the most important control problems in almost every robotic system. This requires a manipulator to move along a specified trajectory by minimizing the tracking error. For parallel manipulators, achieving desired trajectory tracking is difficult due to the combination of nonlinearities, uncertainties, and external disturbances [9]–[11]. Especially in high-precision systems, even small errors in tracking a trajectory can result in a large loss in performance. The most recent advancements in research have focused on the increasing the performance and robustness of the system by optimizing the trajectory tracking techniques [12]–[15].
Due to their ease of use and implementation, the Proportional-Integral-Derivative (PID) Controller is one of the most common control strategies used in the industry [16], [17]. PID controllers adjust system behavior to error by implementing proportional, integral, and derivative actions. PID controllers are limited when used with nonlinear systems like parallel manipulators. PID controllers, in the case of changing working conditions, show poor robustness, are slow to converge, and have large overshoot [18]–[20]. Traditional PID controllers have been shown to require innovative tuning methods for the nonlinear dynamics and uncertainty case (of PID controllers) [21]–[23].

To address these drawbacks, newer control methods, like the Linear Quadratic Regulator (LQR), have been employed [24], [25]. LQR uses a cost function that is quadratic in nature and allows a balance between performance of the system and the control effort for an optimal control solution. Because of the nature of state feedback control [26], LQR control is optimal and bounded. Unlike LQR, in trajectory tracking even the control is bounded, steady state error and/or disturbances must be mitigated [27].
LQR is used as an enhancement or tuning of PID to allow hybrid LQR–PID controllers add better control, stability, and tracking performance [28]–[30]. Good hybrid control performance requires the appropriate tuning of LQR weighting matrices as well as the PID gains. Using optimization methods to hybrid control design has been shown to improve control tracking and system stability performance [31]–[33].
Traditional tuning methods such as Ziegler-Nichols, Cohen-Coon, and trial-and-error approaches do not work well for tuning LQR-PID controllers in nonlinear systems [34], [35]. These methods often fail to deliver the best performance due to the complexity and varied nature of the optimization problem [36]. As a result, evolutionary optimization algorithms have gained a lot of attention for tuning controllers. Genetic Algorithm (GA) and Particle Swarm Optimization (PSO) are among the most commonly used evolutionary algorithms for this purpose [37]-[40].
The Genetic Algorithm (GA) and the Particle Swarm Optimization (PSO) are some of the most widely used evolutionary algorithms for optimal control design [37]–[40]. Although GA has good global search ability, it is known to suffer from the problem of slow convergence, while PSO has fast convergence, but can get stuck in local optima [41], [42]. In order to improve the aforementioned weaknesses, several modern optimization algorithms have been introduced.
Differential Evolution (DE) is a powerful method for optimizing globally and stably [43]. WOA and FA help to optimize through improving the characteristics of exploration and exploitation [44], [45]. BA and SCA offer a good process for searching that emphasizes both exploration and exploitation simultaneously [46], [47]. ABC and BBO are other popular methods for solving optimization problems [48], [49]. In the recent years, a great number of innovations in the field of meta-heuristics have helped increase the rate of convergence significantly [50]–[52].
Recent research has focused on the Grey Wolf Optimizer (GWO), owing to its efficient exploration and exploitation abilities [53]-[55]. This algorithm mimics the social structure and hunting strategies of grey wolves, leading to improved convergent behaviors. Several researchers have illustrated the superiority of GWO in optimization and control applications [56]-[58]. Besides, recent findings have shown that optimal control schemes can significantly increase the performance of robots and parallel manipulators [59]. However, despite all these features, GWO is underexploited when applied to tune the LQR-PID controllers in trajectory tracking of 3-DOF parallel manipulators.

Overview of 3-DOF Parallel Manipulator

A 3-DOF parallel manipulator is made up of a fixed base and a movable platform joined through three separate kinematic chains [1]-[5]. Common forms of such manipulators include 3-RRR and 3-PRS arrangements. Each kinematic chain plays its part in the movements of the end-effector, resulting in a closed-loop system, which increases the rigidity and accuracy of the system.
The major strength of parallel manipulators is load-sharing among several chains leading to increased rigidity and dynamic performance [6]-[8]. Nevertheless, the presence of a closed-loop structure results in

complicated kinematic and dynamic properties. Forward kinematics problem is quite nonlinear; while the inverse kinematics problem, though not complicated, has several solutions [9],[10].

Dynamic analysis of a 3-DOF parallel manipulator includes several coupling terms like inertia, Coriolis and gravity terms between different links [11]-[13]. Such couplings result in difficulty in designing the control system, especially in tracking a desired trajectory. Moreover, external disturbances, frictions and parameter uncertainties add to the challenge [14],[15].

Several advanced approaches like adaptive control, robust control, and optimization-based control have been suggested to overcome such difficulties [16]-[18].To cope with these issues, people have proposed advanced control approaches, like adaptive regulation, robust feedback control, and optimization driven control [16]–[18]. Also, more recent work indicates that optimization based methods can really raise tracking precision and robustness for parallel manipulators [48]–[50].

Mathematical Formulation of LQR–PID Controller

The Linear Quadratic Regulator (LQR) is an optimal control technique that determines the control input by minimizing a quadratic cost function:

$$J(u)=\int_0^\infty [x^T(t)Qx(t) + u^T(t)Ru(t)]dt \quad (1)$$

Where Q and R are positive semi-definite and positive definite weighting matrices, respectively [24]–[26].

The state feedback control law that minimizes the above quadratic objective function is written below:

$$u(t)= -R^{-1}B^TPx(t)= -Fx(t) \quad (2)$$

Where, state feedback gain (F) = $R^{-1}B^TP$ (3)

and $P = \left[P_{11}\, P_{12}\, P_{13}\, P_{12}\, P_{22}\, P_{23}\, P_{13}\, P_{23}\, P_{33}\right]$, P represents the symmetric positive matrix, it can be obtained by feeding the appropriate values of Q and R matrices in the Algebraic Riccati equation (14), as written below,

$$A^TP + PA - PBR^{-1}B^TP + Q = 0 \quad (4)$$

Once P matrix is obtained then, after substitute it in the state feedback gain function (3), as written below;

$$F= -R^{-1}B^TP= -R^{-1}[0\ 0\ K\,]\left[P_{11}\, P_{12}\, P_{13}\, P_{12}\, P_{22}\, P_{23}\, P_{13}\, P_{23}\, P_{33}\right] \quad (5)$$

and $F = -R^{-1}K\left[P_{13}\, P_{23}\, P_{33}\right]=\left[K_i\, K_p\, K_d\right]$ (6)

From above equation the PID gain parameters are computed.

Such that $K_i= -R^{-1}KP_{13}$, $K_p= -R^{-1}KP_{23}$ $K_d= -R^{-1}KP_{33}$ (7)

Where, K_p, K_i, and K_d represent proportional, integral, and derivative gains [16], [17].

In the hybrid LQR–PID framework, LQR is used to compute optimal feedback gains, while PID provides error compensation and disturbance rejection [28]–[30]. This combination improves system stability, reduces steady-state error, and enhances transient performance. Recent studies have demonstrated that optimization-based tuning of LQR–PID controllers significantly improves trajectory tracking performance in nonlinear robotic systems [31]–[33], [51]–[53].

Comparative Study of Evolutionary Algorithms

The tuning of LQR–PID controllers for trajectory tracking of 3-DOF parallel manipulators is a complex nonlinear optimization problem involving multiple conflicting objectives such as minimizing tracking error, reducing overshoot, and improving settling time. Optimization problems frequently fail to satisfy requirements of optimization techniques owing to their multimodal and high-dimensional nature. Thus, evolutionary optimization algorithms have become very popular among researchers for the purpose of tuning control system parameters because of their ability to conduct global searches as well as deal with nonlinearities [27]–[33].

During recent decades, many algorithms based on concepts of evolution as well as swarm intelligence were created and applied to optimize control systems. All these algorithms possess different means to balance between global and local searches. As a result, each of them is characterized by a set of unique features that makes some of them more applicable than others for certain tasks. In this paper, several popular evolutionary optimization algorithms including Genetic Algorithm (GA), Particle Swarm Optimization (PSO), Differential Evolution (DE), Whale Optimization Algorithm (WOA), Firefly Algorithm (FA), Bat Algorithm (BA), Sine Cosine Algorithm (SCA), Artificial Bee Colony (ABC), Biogeography-Based Optimization (BBO), and Grey Wolf Optimizer (GWO) are discussed. The comparative features of these algorithms are provided in Table 1.

Table 1. Comparative Analysis of Evolutionary Algorithms for LQR–PID Tuning

Algorithm	**Key Idea**	**Strengths**	**Weaknesses**	**Ref**
GA	Evolutionary selection	Global search	Slow convergence	[27]
PSO	Swarm intelligence	Fast convergence	Local minima	[30]
DE	Mutation-based	High accuracy	Moderate speed	[33]
WOA	Bubble-net hunting	Good exploration	Stability issues	[34]
FA	Light attraction	High precision	High computation	[35]
BA	Echolocation	Balanced search	Moderate efficiency	[36]
SCA	Mathematical oscillation	Fast exploration	Weak exploitation	[37]
ABC	Bee foraging	Robust search	Slower convergence	[38]
BBO	Migration-based	Strong exploitation	Limited diversity	[39]
GWO	Wolf hierarchy	Robust, balanced	Less explored	[40]

As observed from Table 1, traditional algorithms such as GA and PSO have been extensively used in controller tuning applications due to their simplicity and ease of implementation. Although GA shows good global searching capability, slow convergence makes it less useful in solving problems that require real-time applications. On the other hand, PSO leads to fast convergence, but this method is vulnerable to premature convergence especially when dealing with complicated nonlinear systems [30]–[32].

DE offers a good balance between the disadvantages of both algorithms by improving global optimization performance due to use of mutations and crossover techniques. Likewise, FA guarantees accurate solutions because of an effective attraction-based search process although at the cost of increased computation time [33], [35]. WOA and BA strengthen exploration abilities helping avoid local minimums, but their success can depend on the correct selection of parameters [34], [36].

In ABC, inspiration for optimization lies in foraging activities of bees while BBO draws upon species migration phenomena. The benefits of such approaches lie in increased robustness and ability to exploit potential solutions effectively, but they may be hindered by slow convergence speeds in complex conditions [38], [39]. On the contrary, SCA allows fast exploration of the solution space using oscillating functions, but limited exploitation capability becomes a challenge in adjusting control parameters [37]. Compared to other optimization techniques, GWO strikes an optimal balance between exploration and exploitation. This algorithm makes use of its hierarchical design and hunting process to efficiently traverse through the search space without falling into local optima [40]–[42]. For this reason, GWO is best suited for optimizing LQR-PID controllers of nonlinear processes.

Conclusion and Research Gaps

This paper carries out an extensive comparison of evolutionary algorithms for LQR-PID controller tuning in the trajectory tracking of 3-DOF parallel manipulators. The research highlights the drawbacks associated with existing tuning techniques and illustrates the usefulness of evolutionary algorithms to resolve the nonlinearity problem and improve the controller's performance. Of all the algorithms under consideration, the conventional algorithms, such as GA and PSO, can provide satisfactory results but suffer from poor convergence rate and early convergence. On the other hand, the advanced algorithms include DE, FA, WOA, and their variants that can perform better in terms of optimization but require additional resources or exhibit a high degree of sensitivity.

Notably, GWO deserves special attention owing to the effective balance of exploration and exploitation operations leading to faster convergence and better performance. Unfortunately, there is still little research conducted in the application of GWO to the tuning of LQR-PID controllers of 3-DOF parallel manipulators. The identified research gap provides an opportunity for future studies that will focus on GWO and hybrid optimization algorithms to improve the performance and robustness of trajectory tracking in parallel manipulators.

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