Image Encryption using Seven-Dimensional Logistic-Sine-Tan (7D-LST) Hyperchaotic Map

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Abstract: In chaos-based image encryption, it is desired that the pseudorandom source has to survive simultaneously to statistical, differential and brute-force attacks with the aid of dynamical complexity, sensitivity to initial conditions and key space. The limited key space and narrow chaotic windows in low-dimensional chaotic map inspire the construction of chaotic map in high dimension. This paper proposes a new seven dimensional hyperchaotic Logistic–Sine–Tan (7D-LST) map, which interleaves three nonlinear kernels – logistic, sinusoidal and hyperbolic tangent – in a repeating sequence L→S→T, and has two propagation connecting coefficients γ, δ between any nonlinear kernel that can be bidirectional, causing the map to create a circulant coupling graph between the nonlinear kernels that are not adjacent. Thus, the situation lacks periodicity in the vicinity of a fold point and corresponds to a larger phase-space mixing as compared to a homogeneous logistic or sine system. The key space for the 640-bit secret key (eight initial states, the control parameter μ and two values for the coupling weights) is $2^{640} \approx 4.6\times10^{192}$, which is much larger than 2^{128}, the AES-256 key space. These properties make the 7D-LST map a good pseudo random generator in image high-security encryption.

Keywords: Image Encryption; High dimensional; Logistic Map; Sine Map; Tan Map etc.

Introduction

In this work, a new proposed hyperchaotic map, named 7D Logistic-Sine-Tan (7D-LST) map, is added. It is not a published system before the work, but it is a new design concept inspired by the cross-coupling design philosophy of the lower-dimensional hybrid mapping system [1][2]. The key innovation is the structured rotation between three different nonlinear kernels (called logistic quadratic (L), sinusoidal (S) and hyperbolic tangent (T)) being rotated with three different heterodox state variables (L→S→T→L→S→T→L) with bidirectional cross-coupling terms between non-adjacent variables.

With this design, you'll get three objectives in a single sport! First, by three different nonlinear functions, the periodic braided orbit gets structurally asymmetric, unlike the pure logistic or pure sine maps, that have certain braided residue of periodicity near fold points. Second, the cross-coupling is 7 dimensional, involving two coupling coefficients (γ and δ) which leads to a rich cross coupling graph, where any state variable influences and is influenced by 3 kernel family of state variables. Thirdly, the sensitivity to initial conditions of the hyperbolic tangent function is more aggressive than both the sinusoidal function and the logistic function, and yields the highest value in the sum of the aggregate Lyapunov exponents for all four 7D maps studied in this work.

Proposed Methodology

The 7D-LST map is defined by the following system of seven coupled difference equations:

$$\begin{aligned}
x_1(n+1) &= \{\, \mu\cdot x_1(1-x_1) \;+\; \gamma\cdot\sin(\mu\pi x_2) \;+\; \delta\cdot\tanh(\mu\cdot x_3)\,\} \bmod 1 \\
x_2(n+1) &= \{\, \sin(\mu\pi x_2) \;+\; \gamma\cdot\tanh(\mu\cdot x_4) \;+\; \delta\cdot\mu\cdot x_5(1-x_5)\,\} \bmod 1 \\
x_3(n+1) &= \{\, \tanh(\mu\cdot x_3) \;+\; \gamma\cdot\mu\cdot x_6(1-x_6) \;+\; \delta\cdot\sin(\mu\pi x_7)\,\} \bmod 1 \\
x_4(n+1) &= \{\, \mu\cdot x_4(1-x_4) \;+\; \gamma\cdot\sin(\mu\pi x_5) \;+\; \delta\cdot\tanh(\mu\cdot x_6)\,\} \bmod 1 \\
x_5(n+1) &= \{\, \sin(\mu\pi x_5) \;+\; \gamma\cdot\tanh(\mu\cdot x_7) \;+\; \delta\cdot\mu\cdot x_1(1-x_1)\,\} \bmod 1 \\
x_6(n+1) &= \{\, \tanh(\mu\cdot x_6) \;+\; \gamma\cdot\mu\cdot x_2(1-x_2) \;+\; \delta\cdot\sin(\mu\pi x_3)\,\} \bmod 1 \\
x_7(n+1) &= \{\, \mu\cdot x_7(1-x_7) \;+\; \gamma\cdot\sin(\mu\pi x_1) \;+\; \delta\cdot\tanh(\mu\cdot x_4)\,\} \bmod 1
\end{aligned} \tag{1}$$

where {u} mod 1 denotes the fractional part of u, i.e., {u} mod 1 = u − ⌊u⌋, which maps the result to [0, 1) after each iteration. Conceptually, this is the same as the "modular addition" a $\oplus$ b = {a + b} mod 1 that was introduced in the compact notation.

If we write L(x) = μ·x(1−x), S(x) = sin(μπx) and T(x) = tanh(μ·x), then the system (1) can be compactly expressed as:

$$\begin{aligned}
x_1(n+1) &= \{\, L(x_1) \;+\; \gamma\cdot S(x_2) \;+\; \delta\cdot T(x_3)\,\} \bmod 1 \\
x_2(n+1) &= \{\, S(x_2) \;+\; \gamma\cdot T(x_4) \;+\; \delta\cdot L(x_5)\,\} \bmod 1 \\
x_3(n+1) &= \{\, T(x_3) \;+\; \gamma\cdot L(x_6) \;+\; \delta\cdot S(x_7)\,\} \bmod 1 \\
x_4(n+1) &= \{\, L(x_4) \;+\; \gamma\cdot S(x_5) \;+\; \delta\cdot T(x_6)\,\} \bmod 1 \\
x_5(n+1) &= \{\, S(x_5) \;+\; \gamma\cdot T(x_7) \;+\; \delta\cdot L(x_1)\,\} \bmod 1 \\
x_6(n+1) &= \{\, T(x_6) \;+\; \gamma\cdot L(x_2) \;+\; \delta\cdot S(x_3)\,\} \bmod 1 \\
x_7(n+1) &= \{\, L(x_7) \;+\; \gamma\cdot S(x_1) \;+\; \delta\cdot T(x_4)\,\} \bmod 1
\end{aligned} \tag{2}$$

Rotation of primary kernels is: L, S, T, L, S, T, L. One primary kernel and two cross-coupling terms from the other two kernel families are used for each of the variables. The cross-coupling index offsets are +1 and +2 (mod 7), respectively, where a circulant coupling structure is formed. Tanh has a range of (−1, 1). If one of the state variables leaves the range (0, 1) during iteration, a post-step normalisation is taken to ensure that all the state variables stay in this range during iteration.

$$x_i(n+1) \leftarrow (1 + \tanh(x_i(n+1))) / 2 \quad \text{if } x_i(n+1) \notin (0,1) \tag{3}$$

For properly selected parameters bounds (γ, δ ≤ 0.05) and initial conditions $x_i(0) \in (0.1, 0.9)$ of the modular reduction in Eq. (2) an itself is enough to keep iterates in [0, 1).

The output sequences after discarding first N_t = 1000 transient iterates are obtained, as interleaved sequences of all the seven state variables, with following proper order:

$$K = \{\, x_1(1000), x_2(1000), \ldots, x_7(1000), x_1(1001), x_2(1001), \ldots \,\} \tag{4}$$

To get the keystream bytes for use in the diffusion stage of the encryption algorithm, each floating-point value is quantized to an 8-bit integer, $k_n = \lfloor 256 \cdot K_n \rfloor \bmod 256$.

Results

The encryption was done based on the 7D LST hyperchaotic map whose hyperchaotic parameters were verified. Figure 2 and 3 illustrated the simulation visual results and their histogram.

The proposed 7D-LST chaotic map uses seven initial state values $x_1(0)$, $x_2(0)$, $x_3(0)$, $x_4(0)$, $x_5(0)$, $x_6(0)$, and $x_7(0)$ as the secret key. Each of which are represented by 64 bits, adding up to 7×64 = 448 bits. The system also has a 64-bit complex input parameter and two 64-bit complex coupling parameters γ and δ, which provide 128 bits. Thus, the overall secret key size is 448+64+128=640 bits. So the total key space of the proposed cryptographic scheme is 2^{640}, which is approximately 4.15×10192. This far exceeds the widely adopted minimum threshold of $2^{128} \approx 10^{38}$ required for resistance against brute-force attacks, and is substantially larger than AES-256 (2^{256}). The security of the proposed scheme is therefore a high level of cryptographic security to protect from exhaustive key-search attacks. Table 1 compares the key space of the proposed 7D-LST hyperchaotic image encryption scheme with several recently reported methods.

$$\text{Key Space} = 2^{(64\times10)} = 2^{640} \approx 4.15 \times 10^{192} \quad (4)$$

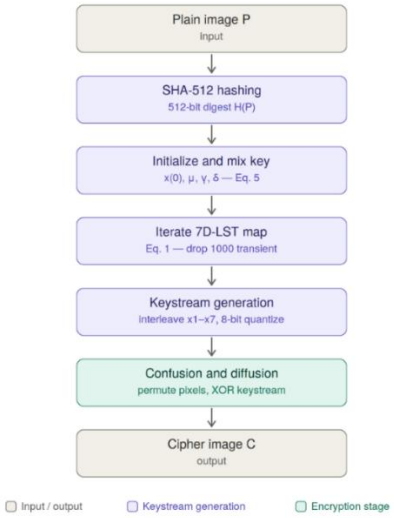

Figure 1. Flow chart of the Proposed Method.

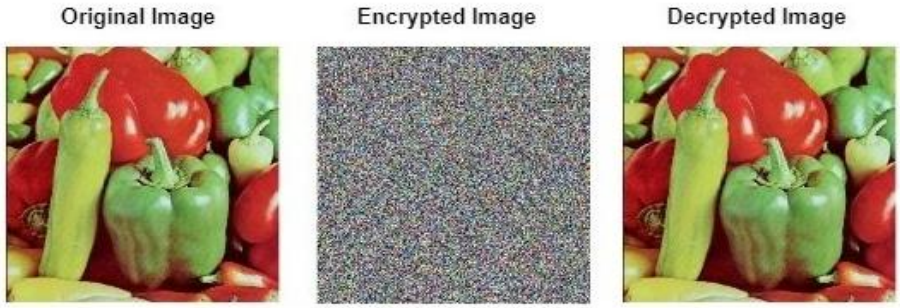

Figure 2. Visual Results.

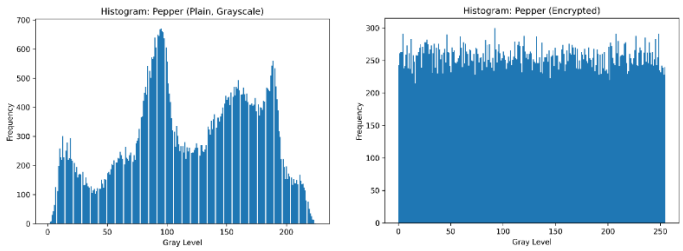

Figure 3. Histogram Analysis.

Table 1. Comparative analysis of key space.

Literature	Key Space
Our 7D-LST	2^{640}
Ref [3]	2^{60}
Ref [4]	2^{299}
Ref [5]	2^{364}

Conclusion

In this work, a new (Logistic–Sine–Tan) 7D hyperchaotic map is proposed, which is an original construction consisting of rotating logistic, sinusoidal and hyperbolic-tangent kernels over seven cross-coupled state variables and implementing modular reduction. From the cryptographic point of view, the 640-bit key space together with the plain-image binding using the SHA-512 hash function gives a total key space of 2^{640} ($\approx 10^{192}$), which is higher than the conventional 2^{128} level of security and the key space of AES-256. Also, due to the hashing, the plain-image space is sensitive (against chosen-ciphertext and known-plaintext attacks). Practical tips on initial-condition bounds (cuts) and weak-key exclusion ($\mu \geq 3.87$) and transient removal (the first 1000 iterates) make sure that the keystream is generated from the fully developed chaotic attractor. These features, taken as a whole, make the 7D-LST map a powerful pseudo-random number generator for image-encryption pipelines.

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